ADVANCES IN ATMOSPHERIC SCIENCES, 2025

• Review •

A Nonlinear Theory and Technology for Reducing the Uncertainty of High-Impact Ocean–Atmosphere Event Prediction※

Mu MU[1] and Wansuo DUAN*[2]

[1]*Department of Atmospheric and Oceanic Sciences/Institute of Atmospheric Sciences, Fudan University, Shanghai* 200438, *China*

[2]*National Key Laboratory of Earth System Numerical Modeling and Application, Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing* 100029, *China*

(Received 13 November 2024; revised 15 January 2025; accepted 5 February 2025)

ABSTRACT

In this article, our nonlinear theory and technology for reducing the uncertainties of high-impact ocean–atmosphere event predictions, with the conditional nonlinear optimal perturbation (CNOP) method as its core, are reviewed, and the "spring predictability barrier" problem for El Nino–Southern Oscillation events and targeted observation issues for tropical cyclone forecasts are taken as two representative examples. Nonlinear theory reveals that initial errors of particular spatial structures, environmental conditions, and nonlinear processes contribute to significant prediction errors, whereas nonlinear technology provides a pioneering approach for reducing observational and forecast errors via targeted observations through the application of the CNOP method. Follow-up research further validates the scientific rigor of the theory in revealing the nonlinear mechanism of significant prediction errors, and relevant practical field campaigns for targeted observations verify the effectiveness of the technology in reducing prediction uncertainties. The CNOP method has achieved international recognition; furthermore, its applications further extend to ensemble forecasts for weather and climate and further enrich the nonlinear technology for reducing prediction uncertainties. It is expected that this nonlinear theory and technology will play a considerably important role in reducing prediction uncertainties for high-impact weather and climate events.

Key words: predictability, optimal perturbation, error growth, targeted observation, ensemble forecast

Citation: Mu, M., and W. S. Duan, 2025: A nonlinear theory and technology for reducing the uncertainty of high-impact ocean–atmosphere event prediction. *Adv. Atmos. Sci.*, https://doi.org/10.1007/s00376-025-4467-9.

Article Highlights:

- A novel nonlinear theory and technology have been introduced aiming at reducing uncertainty in predicting high-impact ocean-atmosphere events.
- The scientific rigor and effectiveness of this theory and technology have been verified through field campaign of targeted observations.
- Future research holds promise for integrating CNOP with artificial intelligence.

1. Introduction

Studies of weather and climate predictability are the basis and key to numerical weather forecasts and climate predictions (Thompson, 1957; Buizza and Palmer, 1995; Mu et al., 2004; Duan et al., 2023a). These studies not only compose the forefront of international scientific endeavors in this research field but also meet major demands such as disaster prevention and reduction. The main focus of predictability studies is to delve into the sources and mechanisms that lead to prediction uncertainties and explore methods and approaches to reduce uncertainties in weather and climate predictions (Mu et al., 2004).

Observational errors are the basic factors causing forecast errors in both weather forecasts and climate predictions. Ever since the pioneering discovery of atmospheric chaos by Lorenz (1963), nonlinearity has been widely recognized to play a crucial role in the rapid amplification of initial (observational) errors, leading to significant forecast uncertainties (Toth and Kalnay, 1997; Mu et al., 2003; Ding and Li, 2007; Duan and Huo, 2016). However, owing to the lack of effective methods for addressing nonlinear challenges, linear methods such as the normal mode (Lord Rayleigh, 1879) and singular vector (SV; Lorenz, 1965) methods have

※ This paper is a contribution to the special topic on The 40th Anniversary of Advances in Atmospheric Sciences and the Centenary of the Chinese Meteorological Society.

* Corresponding author: Wansuo DUAN
Email: duanws@lasg.iap.ac.cn

been employed since the 1990s to investigate the dynamics of initial error growth (Farrell, 1990; Molteni and Palmer, 1993; Palmer et al., 1998). Notably, the SV method, as a key technique in atmospheric predictability studies, has been practically used in operational forecasting. Well-known institutions such as the European Centre for Medium-Range Weather Forecasts have incorporated the SV method into operational weather ensemble forecasts (Mureau et al., 1993; Buizza and Palmer, 1995; Molteni et al., 1996; see the link https://confluence.ecmwf.int/display/FUG/). The World Meteorological Organization conducted targeted observation experiments via the SV method for tropical cyclone (TC) forecasting within the international Atmospheric Science Program "Observing System Research and Predictability Experiment" from 2005 to 2014 (THORPEX; WHO, 2011). This collective effort has produced a linear theory and techniques, with the SV method as its core, aiming to effectively reduce forecast errors.

The inherent limitations of linear theory hinder its ability to reveal the nonlinear mechanisms underlying substantial prediction errors in high-impact ocean–atmosphere events; thus, corresponding linear techniques that reduce observational errors through targeted observations and mitigate prediction uncertainties via ensemble forecasts face the challenge of significantly increasing the efficacy of prediction error reduction [see the review by Duan et al. (2023a)].

In view of the limitations of SV-based linear theory and techniques, Mu et al. (2003) fully considered nonlinear physical processes and proposed the conditional nonlinear optimal perturbation (CNOP) method, which generalizes the optimal perturbation method from the linear to nonlinear regime. The CNOP represents the initial error that satisfies a certain physical constraint and causes the largest prediction error. Considering the effects of model parameters on prediction uncertainties, Mu et al. (2010) further upgraded the CNOP to disclose the optimal combination of initial and parametric errors that produces the largest prediction error [also see Duan and Zhang (2010)]. The applications of CNOPs subsequently extended to investigations of the predictability of the El Niño–Southern Oscillation (ENSO) and targeted observations for TC forecasting (Mu et al., 2007b, 2007c, 2009; Duan and Wei, 2013), thereby establishing a new theory to address significant forecast errors arising from nonlinearities, environmental conditions, and initial errors of specific spatial patterns. Moreover, a novel method to identify sensitive areas for targeted observations, which is based on CNOP spatial patterns and environmental conditions, was proposed (Mu et al., 2009, 2015b). This method has been validated in practical field campaigns for targeted observations associated with TC forecasts and ocean state predictions, and a new technology has since been established that uses CNOP for targeted observation strategies to reduce observational and forecast errors. This collective effort has surpassed the SV-based linear theory and technique and established a new theory and technology, with the CNOP as its core, that fully considers nonlinear effects and provides a scientific foundation and a technical approach to improve the prediction accuracy of high-impact ocean–atmosphere events.

This article reviews this nonlinear theory and technology and examines its international recognition. In the following section, the CNOP method and its calculation are introduced; simultaneously, the recognition level of the CNOP is provided. In section 3, the nonlinear theory is interpreted, and associated comments are reviewed; then, in section 4, the nonlinear technology, together with relevant field campaigns, for reducing observational errors and forecast uncertainties is elucidated. Additional research on the CNOP method and its applications is also introduced to support nonlinear theory and technology in section 5; finally, a summary and prospects are provided in section 6.

2. The method: conditional nonlinear optimal perturbation

The SV method has played an important role in atmospheric predictability studies, offering valuable insights into weather forecasting and climate prediction (Buizza and Palmer, 1995; Molteni et al., 1996; Moore and Kleeman, 1997; Palmer et al., 1998). However, Mu (2000) revealed that SV is the optimal perturbation only when initial errors are sufficiently small and/or the forecast time period is short, which is an unrealistic scenario for real-world predictions; moreover, in practical forecasting, it remains a challenge to determine whether initial errors are sufficiently small and/or if the forecast time is short enough. In addition, the SV is obtained by calculating the maximum ratio of linear development of the initial perturbation to the initial perturbation and has to represent the direction of initial perturbation growth, ultimately being unable to capture the full impact of finite-amplitude errors on the prediction results (Mu et al., 2003; Harle et al., 2006). Therefore, the SV is limited to characterizing the initial error that has the largest effect on the prediction results under linear assumptions. These limitations require methodological breakthroughs within the nonlinear regime.

2.1. *Conditional nonlinear optimal perturbation*

Mu et al. (2002) designed a nonlinear optimization problem aimed at revealing the largest prediction error by using observational error information as a physical constraint for initial errors and defining the maximum nonlinear development of the initial errors as the objective function. On the basis of this nonlinear optimization problem, Mu et al. (2003) proposed the CNOP method for exploring optimally growing initial errors in the nonlinear regime by considering the constraints of the SV method mentioned above. Practically, the predictions are influenced by both initial errors and model errors, as well as their interactions. In terms of this point, Mu et al. (2010) further extended the CNOP method to cover optimal combinations of initial and model parameter perturbations [also see Duan and Zhang (2010)]. This extension increased the ability of the CNOP method to explore not only the effects of the initial error but also the effects of the model parameter errors, including their nonlinear coupling effects. The upgraded CNOP method includes two special cases: CNOP-I and CNOP-P. The former is the CNOP under the perfect model scenario in Mu et al. (2003), and the latter is for the optimal parametric perturbation under the perfect initial condition assumption. The upgraded CNOP method can identify the relative importance of initial errors and model parameter errors and has subsequently been applied to predictability studies of ENSO (Duan and Zhang, 2010; Yu et al., 2012). They showed that initial errors with particular spatial patterns are more likely than model parametric errors to result in larger forecast errors of ENSO events. This finding provides a theoretical foundation for decreasing initial errors and improving ENSO prediction accuracy (see section 3).

Many studies have confirmed that the CNOP method fully incorporates the effects of nonlinearity and transcends the linear constraints inherent in the SV method via dynamic analysis and/or numerical experiments in predictability studies of ENSO, TC, and other high-impact ocean–atmosphere events [see the reviews by Duan and Mu (2009) and Duan et al. (2023a, 2023b)]. Moreover, Magnusson et al. (2008) suggested that the CNOP method finds the most unstable perturbations by maximizing the perturbation via the nonlinear model instead of the tangent linear model adopted by the SV. Winkler et al. (2020) reported that the SV method misses important nonlinear developments and that the CNOP method overcomes this limitation. Harle et al. (2006) suggested that, compared with the SV method, the CNOP method finds initial perturbations that will most likely lead to a certain future state. Obviously, CNOP represents the optimal perturbation when nonlinear processes in the model are fully considered.

Terwisscha van Scheltinga and Dijkstra (2008) noted that the CNOP of a steady state determines the dominant time-dependent nonlinear behavior of finite amplitude perturbations, and such behavior bridges the gap between the behavior below the energy stability boundary and the linear stability boundary; they also noted that, compared with nonnormal modes, the CNOP displays how much nonlinearity affects the evolution of finite amplitude perturbations, and for the case of linearly stable multiple equilibrium, the CNOP method provides a way to compute finite amplitude stability boundaries of each equilibrium. From this perspective, the CNOP method is also more advanced than linear methods such as SV in terms of its ability to assess the importance of nonlinearity and related additional contributions to the understanding of weather and climate stability and related predictability.

Because of the progressiveness of the CNOP method in predictability studies, Terwisscha van Scheltinga and Dijkstra (2008) used the CNOP method to reveal the predictability of the oceanic double gyre. Rivière et al. (2009) adopted it to explore the error growth dynamics of the atmospheric moisture process. Wang et al. (2020a) extended it to operational forecast models for convective-scale ensemble forecasts. In a review of the key progress in meteorological studies in China provided by Shen et al. (2020), the authors reported that the CNOP method has been applied to a broad range of predictability research at various timescales, targeted observations, and ensemble forecasts.

2.2. *Computation of the conditional nonlinear optimal perturbation*

Numerical computation of CNOPs is a challenge in the context of predictability study applications. Mu et al. (2003) addressed this challenge by providing theoretical derivations of the adjoint algorithm for computing CNOP; subsequently, a CNOP optimization scheme for high-dimensional numerical models was formed by integrating the advanced optimal algorithm, e.g., the spectral projected gradient algorithm (SPG2; Birgin et al., 2000), with a high-performance computing cluster. The CNOP optimization system was established for well-known models such as the Zebiak–Cane model for ENSO (10^4 dimensions; Mu et al., 2007b) and the Mesoscale Model 5 (MM5, 10^5 dimensions; Mu et al., 2007a, 2009). For clarity, we plot in Fig. 1 a diagram outlining the CNOP computation process. In subsequent studies, the CNOP optimization system was further constructed for more realistic models, such as the Weather Research and Forecasting model (WRF; 10^5 dimensions) for the atmosphere and Regional Ocean Modeling System (ROMS; 10^7 dimensions) for the ocean [see the review by Wang et al. (2020b)]. This algorithmic foundation has paved the way for applying the CNOP method to predictability studies related to high-impact weather and climate events. Birgin et al. (2014) recognized that the CNOP method is a real application of the SPG2 algorithm.

In addition to the above adjoint gradient algorithm, several studies have explored adjoint-free gradient algorithms for calculating CNOP. Wang and Tan (2010) developed an ensemble-based adjoint-free gradient algorithm in atmospheric predictability research. Shi and Sun (2023) performed it via a sampling method in the field of mathematics. Tian and Feng (2017) presented a nonlinear least-squares-based algorithm to calculate CNOP in studies of atmospheric dynamics. Moreover, some studies have investigated gradient-free algorithms to solve CNOPs. Oosterwijk et al. (2017) proposed using principal component analysis for dimension reduction and then performing optimization to calculate CNOP for studying oceanic circulation predictability. Zhang et al. (2019) developed a modified direct search algorithm based on a kernel density estimator to solve CNOP in the field of software. Peng and Sun (2014) introduced projection skill to the differential evolution algorithm for calculating CNOP to study land surface predictability. In addition, many intelligent algorithms have been introduced to solve CNOPs (Mu et al., 2019; Zhang et al., 2018; Yuan et al., 2022, 2023; Mu et al., 2015a). In any case, the new algorithms contribute to successful applications of the CNOP method in complex problems in various research fields and enhance the extensiveness of CNOP applications.

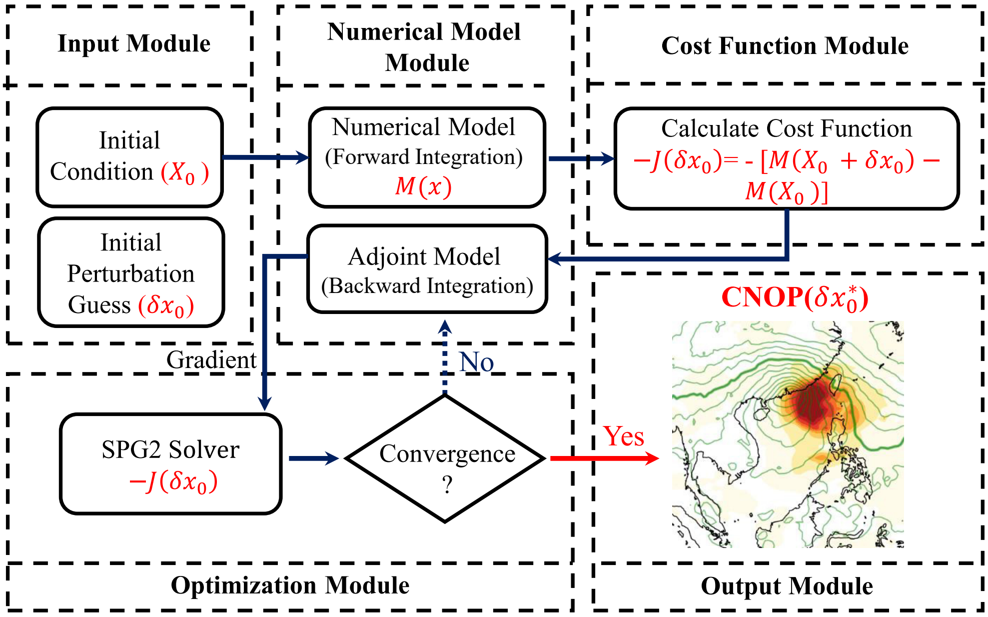

Fig. 1. Diagram showing the CNOP computation process.

As reviewed above, the CNOP method was first proposed for exploring optimally growing initial perturbations in nonlinear models; subsequently, it was further generalized to reveal the optimal combined mode of initial and model parametric perturbations. Notably, in the field of fluid mechanics, a similar optimal perturbation method, which is applicable only for unraveling the optimal initial perturbation, i.e., nonlinear optimal perturbation (NLOP; Pringle and Kerswell, 2010), was also proposed seven years after that of the CNOP method. To date, the NLOP method has been applied to research fields such as the stability of fluid mechanics, acoustics, and magnetics [see the review by Kerswell et al. (2014)]. In particular, Kerswell et al. (2014) recognized that the NLOP method is a type of CNOP method. The CNOP methodology is highly important for exploring the stability and predictability of various field dynamics and, since it was first proposed, has triggered interdisciplinary research and achieved great international attention.

3. A nonlinear theory for prediction error growth dynamics achieved via the CNOP method

High-impact ocean–atmosphere events severely influence the human living environment. Their onset, development and demise are generally related to the interactions between the ocean and atmosphere and the corresponding dynamic and thermodynamic processes. The types of events often have different spatial and temporal scales. For example, ENSO events, which occur in the tropical Pacific and present interannual variability, and TC events, which are powerful and profound tropical weather systems, are low-pressure vortexes that occur over tropical or subtropical oceans. The CNOP method has been applied to predictability studies of these two types of high-impact weather and climate events, and a nonlinear theory for error growth in high-impact ocean–atmosphere event predictions has been established (Fig. 2). This section introduces this theory and reviews its role in understanding the predictability of high-impact ocean–atmosphere events.

3.1. *Nonlinear mechanism of the "spring predictability barrier" for ENSO events*

The "spring predictability barrier" (SPB) phenomenon, which refers to a rapid decline in prediction accuracy during boreal spring when ENSO events are predicted (Webster and Yang, 1992; Duan and Wei, 2013), aggressively limits the ENSO prediction level. Owing to its complex and elusive nature, the SPB has persistently troubled both the atmospheric and oceanic scientific communities for a long time [see the review by Duan and Mu (2018)]. The study of the SPB is a highly challenging frontier in the study of ENSO predictability.

Clarifying the primary sources of prediction errors associated with the SPB and unraveling the mechanisms behind the growth of these errors hold significant scientific and practical value for improving ENSO prediction accuracy (Tang et al., 2018). In SPB research, many studies have relied on the SV method to explore error growth dynamics related to the SPB (e.g., Thompson, 1957; Moore and Kleeman, 1996; Xue et al., 1997; Samelson and Tziperman, 2001), in which they described the SPB as exhibiting the maximum amplification of seasonal-prediction initial errors occurring in the boreal spring. However, for this scenario, the influence of nonlinear physical processes on the SPB was ignored, and the nonlinear mechanisms underlying SPB occurrence were not revealed. In view of this oversight, Mu et al. (2007c), utilizing the CNOP method, first provided a new description of the SPB; specifically, they regarded a significant SPB as

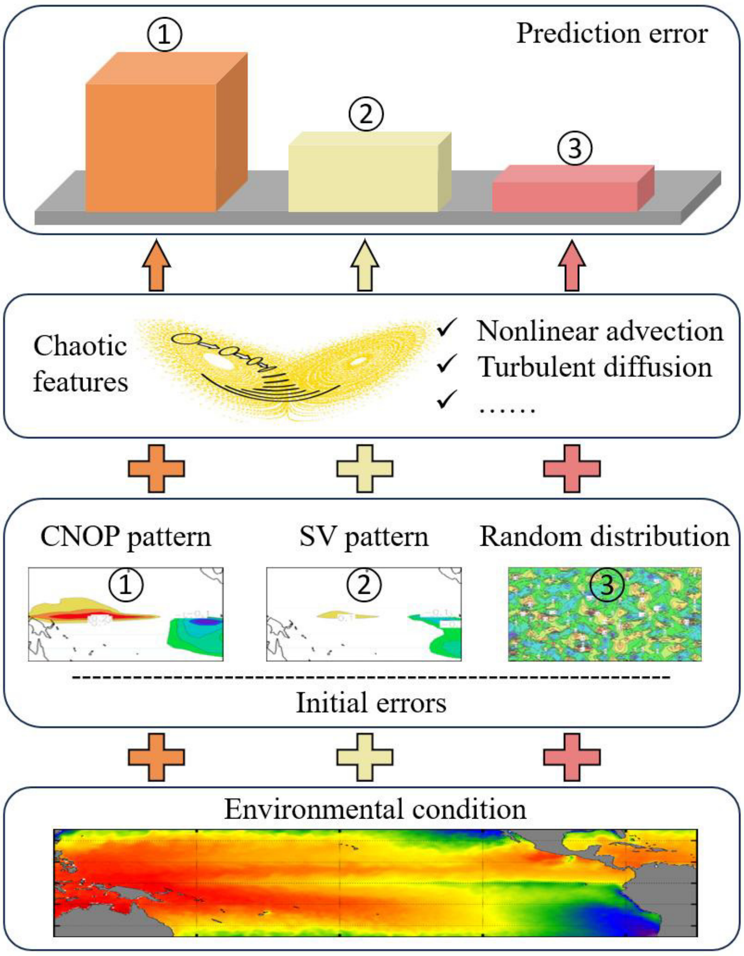

Fig. 2. Sketch map of the nonlinear theory for significant error growth of high-impact ocean–atmosphere events.

the phenomenon in which ENSO prediction has a large prediction error, and in particular, prominent error growth occurs during the boreal spring when the prediction is made before and throughout the spring [also see Duan and Wei (2013)]. This new description comprehensively considers the impact of all nonlinear physical processes on the SPB.

On the basis of the above new description of the SPB, Mu et al. (2007b) and subsequent relevant studies (Duan et al., 2009; Yu et al., 2009) employed the CNOP method to explore the SPB occurrence mechanism, and they additionally emphasized the important role of CNOP error patterns in the occurrence of the significant SPB, whereas previous studies focused mostly on the relationship between ENSO events and/or the climatological seasonal cycle and SPB, although they sometimes used the SV method (Moore and Kleeman, 1996; Chen et al., 1997; van Oldenborgh et al., 1999; Samelson and Tziperman, 2001). Specifically, they revealed the importance of the initial errors of specific spatial patterns, as opposed to the initial errors with a randomly distributed spatial pattern and not aligned with the linear growth direction of initial errors characterized by the SV method, in leading to significant prediction errors of ENSO events (Mu et al., 2007b; Duan et al., 2009). They also elucidated the crucial impact of nonlinear temperature advection in the tropical Pacific on significant SPBs. On the basis of these findings, a nonlinear mechanism for SPB occurrence was therefore proposed: the combined effect of CNOP-type errors, climatological seasonal cycles and ENSO events, as well as the influence of nonlinear temperature advection in the tropical Pacific, lead to the most significant manifestation of the SPB phenomenon (Mu et al., 2007c; Duan and Wei, 2013). To date, a detailed depiction of the SPB occurrence mechanism has been provided in the regime of nonlinear dynamics for ENSO events.

Kramer and Dijkstra (2013) approved the above nonlinear mechanism of the SPB and noted that the mechanism additionally emphasizes the role of the initial error pattern, particularly its interaction with the annual cycle and ENSO cycle, when they reviewed the impact of the climatological seasonal cycle or ENSO phases on the SPB explored in previous studies. Levine and McPhaden (2015) compared the above nonlinear mechanism of the SPB with their stochastic mechanism and suggested that the two mechanisms are complementary: one explores the importance of the initial error pattern, whereas the other examines the role of stochastic external forcing. The highlight of the nonlinear mechanism for the SPB was the effect of a special initial error pattern on the SPB, which indicates that the SPB can be greatly reduced by optimizing the ENSO observation network according to the CNOP error pattern or performing ensemble forecasts considering initial uncertainties estimated by the CNOP method. Huang et al. (2019) reported that the discovery of the specific spatial structure of the initial error provides a new idea for improving the ENSO prediction technique. Clearly, the nonlinear mechanism of the SPB for ENSO has acquired international recognition.

3.2. *Nonlinear mechanism of the optimally growing initial error of TC track forecasting*

TCs are among the most catastrophic weather conditions and pose a significant threat to China, highlighting the urgent need for accurate forecasts to increase disaster prevention and mitigation efforts. TCs originate from the ocean. The challenge lies in obtaining sufficient and high-quality atmospheric observations over the ocean, ultimately limiting the improvement of TC forecasting accuracy. To address this, a new observation strategy, known as "targeted observation", has emerged, which focuses on conducting intensive observations within key sensitive areas to optimize resource utilization (Snyder, 1996). The key issue of the targeted observation is to determine the sensitive area for preferentially increasing additional observations (Mu, 2013). Many studies have adopted linear methods such as SV to find the most sensitive initial error and then, from this error, to identify the sensitive areas for targeted observations associated with TC forecasts (Buizza and Montani, 1999; Wu et al., 2007). However, owing to the complexity, nonlinearity, and multiscale nature of TC genesis and development, the SV method may cause deviation in the resulting sensitive areas from actual areas, ultimately decreasing the efficacy of targeted observations in improving the forecast level of TCs (Mu et al., 2009; Yu and Meng, 2016).

In view of the limitations of SV, Mu et al. (2009) comprehensively addressed the impact of nonlinear physical processes on error growth for TC forecasting. They calculated the CNOP errors and revealed the spatial distribution characteristics of initial errors that lead to maximum forecast errors of TC tracks [also see Qin et al. (2013)]. Specifically, the CNOP errors were concentrated in the vortex structure of the TCs, particularly at the juncture with the subtropical high-pressure zone, which clarifies the important roles that TCs and large-scale weather circulation systems play in the amplification of forecast errors in TC forecasting. CNOP errors for TC track forecasting were found to cause much larger forecast errors than corresponding SV errors did; furthermore, nonlinear advection processes dominated the enhancement of nonlinearities on error growth. On the basis of these findings, a nonlinear mechanism for error growth in TC track forecasting is outlined as follows: the combined effect of CNOP-type errors, large-scale weather circulation systems and TCs, and nonlinear advection processes results in the maximum forecast error of TC tracks. With advancements in the Advanced Research version of the WRF model, this nonlinear mechanism for error growth in TC forecasting was further indicated by the CNOP errors of TC track forecasting made by the WRF model (Chen et al., 2013).

From a more general perspective, the nonlinear error growth mechanism for TC track forecasting emphasizes that initial errors of specific spatial patterns, together with environmental fields and nonlinear processes, contribute to significant forecast errors. In fact, the CNOP errors and their interactions with the climatological seasonal cycle and ENSO cycle, which are highlighted in the nonlinear mechanism of the SPB for ENSO events, also elucidate the combined effects of three analogous factors. On the basis of this unified mechanism for ENSO predictions and TC forecasting, a novel nonlinear theory is naturally established, in which initial errors in specific spatial patterns, environmental conditions, and nonlinear processes collectively result in significant prediction errors in high-impact ocean–atmosphere event predictions (see Fig. 2).

In this nonlinear theory, specific spatial patterns of initial errors are critical conditions for significant prediction errors of high-impact ocean–atmosphere events. Vidard et al. (2015) suggested that CNOP-like errors can guide improvements in monitoring networks and allow one to select better targeted observations, and Chen et al. (2018) indicated that CNOP-like errors can help refine tropical Pacific observation systems. From the perspective provided by these studies, nonlinear theory clearly lays a scientific foundation for using targeted observations to filter out CNOP errors, thereby improving the prediction ability of high-impact ocean–atmosphere events.

4. A nonlinear technology of targeted observation utilizing the CNOP method

Traditional observations emphasize understanding phenomena and revealing facts, whereas THORPEX has shifted its focus to forecasting demands, highlighting the pivotal role of targeted observation (see section 3) in improving TC track forecasting ability (WHO, 2011). This observation strategy has evolved into an operational tool in meteorological departments, producing valuable data. Both THORPEX and Chinese Taiwan's meteorological departments have adopted linear methods to determine the sensitive areas that should preferentially intensify observations, where they assumed that the fastest growth behaviors of initial errors are approximately imitated by the linear dynamics provided by SV, ETKF (ensemble transform Kalman filter), or adjoint-based sensitivity (Palmer et al., 1998; Bishop and Toth, 1999; Wu et al., 2007). This limitation affects the operational effectiveness of targeted observations, especially forecasts of anomalies with strong nonlinear effects, such as track anomalies, and the rapid intensification process of TCs.

The nonlinear theory outlined above emphasizes the indispensable role of CNOP errors with particular spatial patterns in causing the largest forecast error. Inspired by this nonlinear theory, Mu et al. (2009) comprehensively considered nonlinear effects and proposed a novel method to identify sensitive areas of targeted observation on the basis of the spatial pattern and geographical location of CNOP errors [also see Mu et al. (2015b)]. Through the implementation of this method, the sensitive areas for quite a few TC cases were identified for targeted observations (Mu et al., 2009; Chen et al., 2013; Qin et al., 2013); furthermore, the results demonstrated that the sensitive areas identified by the CNOP method exhibit greater physical relevance than those determined by the SV method, thereby more effectively improving the accuracy of TC track forecasts. It is obvious that the CNOP method is more applicable than the SV method for identifying sensitive areas for targeted observations associated with TC track forecasting.

The CNOP method has been applied in practical field campaigns for TC forecasts to identify the scanning area of the Fengyun-4A satellite (FY-4A) and the locations of dropsondes. From 2020 to 2022, five field campaigns were implemented by using FY-4A and/or dropsondes (Feng et al., 2022; Chan et al., 2023; Qin et al., 2023). Five TCs, including Higos (202007), Maysak (202009), Chan-Hom (202014), Conson (2022113) and Chanthu (202114), were observed by FY-4A from the China Meteorological Administration (CMA), and the achieved targeted observations reduced the TC track errors by approximately 100 km, which were averaged for three-day forecasts (Feng et al., 2022). Simultaneously, the former three TCs were also observed via dropsondes from the Hong Kong Observatory, and corresponding targeted observations revealed their superiority over all the other dropsondes in improving the forecast level of the TC track and intensity (Qin et al., 2023). In particular, for TC Mulan (202207) from 8 to 10 August 2022 over the South China Sea, the first-ever ground–space–sky field campaign with enhanced observations included GIIRS (Geostationary Interferometric Infrared Sounder) microwave soundings, round-trip radiosondes and aircraft-launched dropsondes conducted through FY-4B; the targeted observation data in the sensitive areas identified by the CNOP method were assimilated in real time into the operational numerical prediction system of the CMA; and the observational and forecast results were presented in the weather discussion of the CMA, which demonstrated that assimilating the additional data collected in this way had a positive impact on TC forecasts for both track and intensity, especially when the maximum wind speed was reduced by 11% and the ability to forecast heavy rain in southern China was improved (Chan et al., 2023).

The CNOP method was also applied to determine the sensitive area of an oceanic field campaign conducted in the summer of 2019 for the prediction of vertical thermal structure in continental shelf seas of the Yellow Sea (Hu et al., 2021; Liu et al., 2021). The targeted observations helped refine the structure of the initial vertical thermal structure, eventually improving the predictions of the vertical thermal structure at a lead time of 7 days and reducing the forecast errors from 2.02°C to 0.88°C (Liu et al., 2021).

The above practical observation experiments revealed that the targeted observations that were obtained on the basis of the spatial pattern and geographical location of CNOP-type errors indeed greatly reduced observational errors and mitigated forecast errors. This process provides a novel nonlinear technique that uses targeted observations featuring CNOP errors to significantly reduce observational and prediction errors for high-impact ocean–atmosphere events (Fig. 3).

5. Follow-up research further validates and enriches nonlinear theory and technology

The applications of CNOP to the SPB for ENSO events and the targeted observations for TC forecasting involve nonlinear theory and technology aimed at reducing observational errors and mitigating forecast errors. The theory and technology highlight the crucial role of the CNOP error in causing significant forecast error and provide a method to identify sensitive areas for targeted observations required for reducing prediction uncertainties. This theory and technology, with the CNOP method as its core, have further been validated and enriched by extensive follow-up research. In this section, we review these advances.

5.1. *Further validation of the nonlinear theory and technology*

The CNOP method has recently been adopted to produce optimally growing initial errors for predictions of oceanic flows in the Kuroshio extension (Geng et al., 2020; Wang et al., 2020c), sea surface height at midlatitudes with a key focus on the role of mesoscale eddies (Jiang et al., 2022, 2023, and 2024), and ocean states in the South China Sea (Li et al., 2014; Liu et al., 2023a), as well as forecasts of high-impact weather events such as heavy rainfall (Yu and Meng, 2016, 2022; Ke et al., 2022, 2023; Zhang and Tian, 2022), southwest vortices (Chen et al., 2021), Ural blocking events (Ma et al., 2022; Gao et al., 2023), extreme cold events in East Asia (Dai et al., 2021; Han et al., 2023; Li et al., 2023), Madden–Julian Oscillation (MJO) events (Wei et al., 2019, 2020), and heavy air pollution events (Yang et al., 2022, 2023). All these optimally growing initial errors confirmed the crucial role of the initial errors featured by the CNOPs in yielding large prediction errors, again illustrating the scientific rigor of the above nonlinear theory; furthermore, the spatial structures and geographical locations of the errors indicated the sensitive areas for targeted observations associated with corresponding high-impact event predictions. Studies have shown that implementing targeted observations in these sensitive areas has the potential to significantly improve forecast levels according to observation system simulation experiments and/or relevant physical interpretations.

Notably, the CNOP method has been applied to design observation paths for underwater mobile platforms and has been shown to be highly effective at sampling sensitive

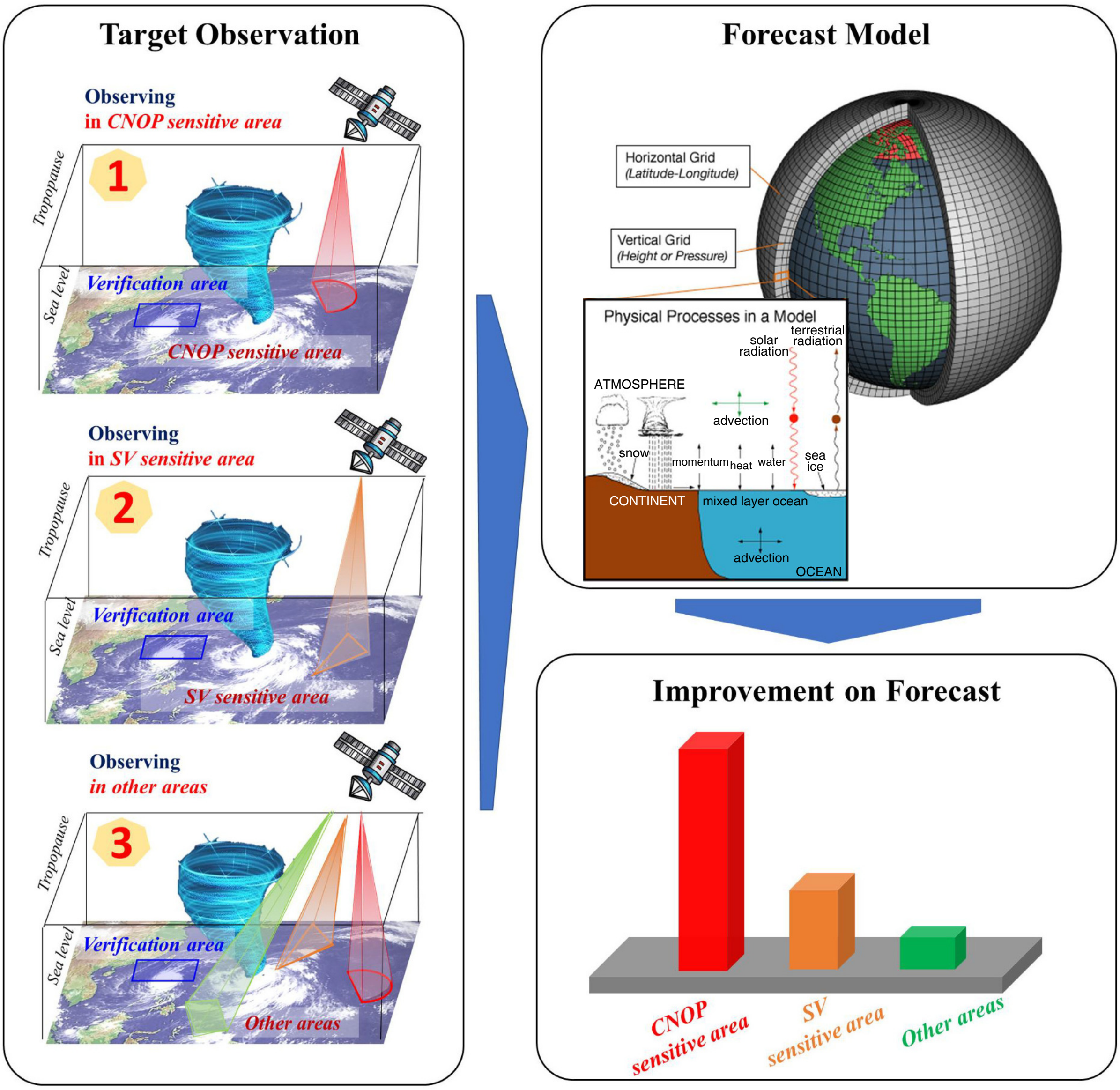

Fig. 3. Sketch diagram of the targeted observation strategy featuring CNOP error.

areas identified by the CNOP method to improve prediction accuracy (Zhao et al., 2023). In addition, the sensitive areas determined by the CNOP method have been extended to design an observing array over the tropical Pacific Ocean with the aim of optimizing the original TAO (Tropical Atmosphere Ocean Array) and further increasing the ENSO forecasting ability (Duan et al., 2018) and to lay out a cost-effective ground meteorological observation-station network for increasing air quality forecasting, resulting in great improvements in the $PM_{2.5}$ concentration forecast ability in the Beijing–Tianjing–Hebei region (Yang et al., 2022, 2023).

Obviously, in follow-up research, either for high-impact atmospheric events or high-impact oceanic events, even high-impact ocean–atmospheric coupling events, nonlinear theory shows its scientific rigor in interpreting why substantial prediction errors occur, whereas its relevant nonlinear technology further illustrates its potential for greatly reducing prediction errors. Thus, nonlinear theory and technology were further validated, and they are expected to play important roles in reducing observational errors and mitigating prediction errors for operational predictions of high-impact ocean–atmosphere events.

5.2. *Further enrichment of nonlinear theory and technology*

The CNOP method originally included special cases of CNOP-I for exploring optimally growing initial errors and CNOP-P for revealing optimal model parametric errors and even covered the optimal combination of initial errors and parametric errors that cause the largest prediction errors (see section 2). Now, it has been further extended to CNOP-B for revealing the boundary uncertainties that have the largest effect on forecasts (Wang and Mu, 2015) and CNOP-F [i.e., the nonlinear forcing singular vector (NFSV) proposed in Duan and Zhou (2013)] for exploring the external forcing errors and the combined effect of various model errors. Thus, a family of CNOP methods has been identified, including CNOP-I, CNOP-P, CNOP-F, and CNOP-B (Fig. 4). All these CNOP methods highlight the significant effects of errors featuring special spatial patterns or composite structures on prediction uncertainties (Wang et al., 2020b) and further enrich the connotation of nonlinear theory.

In addition to nonlinear technology for targeted observations, CNOP methods have recently been further applied to studies of ensemble forecasts. Specifically, the O-CNOPs method, which is based on CNOP-I, was proposed to produce ensemble perturbations for estimating initial uncertainties, with applications to numerical models ranging from the conceptual Lorenz-96 model to the realistic MM5 model and further to the Advanced Research version of the WRF model. These methods have been demonstrated to have the ability to represent the initial error effect and promote the ensemble forecasting ability (Duan and Huo, 2016; Huo et al., 2019; Huo and Duan, 2019). Compared with operationally utilized SVs and BVs (bred vectors), especially for TC track forecasts, the O-CNOPs method provides ensemble members with greater spreads, but they tend to be located on the two sides of real TC tracks and show much better agreement between ensemble spreads and ensemble mean forecast errors (Duan et al., 2023a, 2023b). Furthermore, the O-CNOPs method has been shown to be favorable for reproducing unusual TC tracks in forecasts (Zhang et al., 2023a). For the TC intensity forecasts, the O-NFSVs (also known as "orthogonal CNOP-F") developed from the CNOP-F method were used to depict the effect of model errors, and it was demonstrated that the ensemble members generated by the orthogonal CNOP-F have the ability to represent the model uncertainties affecting TC intensification and to provide much higher ensemble forecasting skills than the operationally employed SPPT (Stochastically Perturbed Parametrization Tendency) and SKEB (Stochastic Kinetic Energy Back-scattering) schemes (Zhang et al., 2023b). For convection-scale weather systems, CNOP-P and CNOP-F methods have been applied to extract more sensitive components or exert more unstable perturbations on SPPT ensembles, consequently increasing the ensemble forecasting ability to a higher level (Wang et al., 2020a; Xu et al., 2022). The CNOP-P has also been adopted to yield ensemble members associated with model parametric perturbations for heatwave event forecasts and achieved satisfying skill (Zhang et al., 2024, 2025).

Therefore, the CNOP method is becoming a new technology for ensemble forecasting. In fact, in the early stage when the CNOP method was proposed, international scholars had already regarded the CNOP method as a new ensemble forecast technology (Harle et al., 2006; Magnusson et al., 2008); also, Mu and Jiang (2008) introduced the CNOP method to SV ensemble forecasting by replacing the leading SV with the CNOP [also refer to Jiang and Mu (2009)] and attempted to improve the related ensemble prediction skill; in recent years, with more successful applications of the CNOP method in ensemble forecasts, especially in realistic ensemble forecasts for ENSO events (see https://soed.sio.org.cn/emsodm.html and http://cmdp.ncc-cma.net/pred/cn_cmme.php?Elem=CMME-ENSO), the CNOP method has been considered an important ensemble forecast method and included in the "Handbook of Hydrometeorological Ensemble Forecasting" published by Springer (Duan et al., 2019). Therefore, the CNOP method, as a new ensemble forecast method, provides a new nonlinear technology that reduces prediction uncertainties for high-impact weather and climate events.

For convenience, we summarize in Table 1 the applications of the CNOP family in follow-up research on high-impact ocean–atmosphere event predictability. Obviously, the applications have extended the CNOP to investigate various scale predictability.

6. Summary and prospects

Linear theory and techniques, with the SV method as its core, hinder their ability to reveal the nonlinear mechanisms responsible for substantial prediction errors in high-impact ocean–atmosphere events and face great challenges in mitigating observational errors and prediction errors for realistic prediction systems. A nonlinear theory and technology, with the CNOP method as its core, was then proposed to understand the mechanisms of significant prediction errors and then to greatly decrease prediction errors for high-impact ocean–atmosphere events.

In this paper, nonlinear theory and technology are reviewed. To overcome the deficiencies of the SV method, the CNOP method was first proposed to fully incorporate the influence of nonlinearity; furthermore, it systematically explores the combined effects of both initial and model parameter errors. Focusing on nonlinearity, high-impact ocean–atmosphere events, and targeted observations, the CNOP

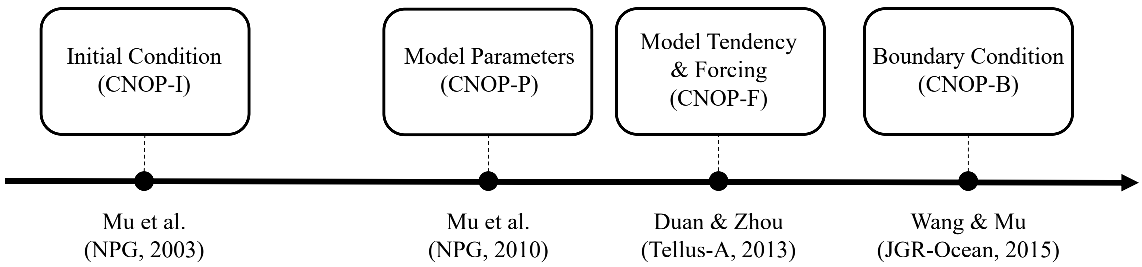

Fig. 4. Evolutionary map for the CNOP family.

Table 1. Applications of CNOP in follow-up research on high-impact ocean–atmosphere event predictability.

	Contents	Key references
Targeted observation	Heavy rainfall forecasts (weather, ~10^0 h)	Yu and Meng (2016, 2022)
	Southwest Vortex forecasts (weather, ~10^0 d)	Chen et al. (2021)
	Atmospheric environment forecasts (weather, ~10^0 d)	Yang et al. (2022, 2023)
	Extended-range forecasts (weather-to-subseasonal, ~10^1 d)	Wang et al. (2014)
	Extreme cold events (S2S, ~10^1 d)	Dai et al. (2021); Han et al. (2023)
	Ural blocking forecasts (S2S, ~10^1 d)	Ma et al. (2022); Gao et al. (2023)
	MJO forecasts (S2S, ~10^1 d)	Wei et al. (2019, 2020)
	ENSO forecasts (seasonal-to-annual, ~10^2 d)	Duan et al. (2018)
	Ocean state predictions (subseasonal, ~10^1 d)	Li et al. (2014); Liu et al. (2023a)
	Kuroshio extension predictions (seasonal, ~10^2 d)	Geng et al. (2020); Wang et al. (2020c)
	Ocean mesoscale eddy predictions (weather, ~10^0 d)	Jiang et al. (2022, 2024)
	Underwater observation optimization design (weather, ~10^0 d)	Zhao et al. (2023)
Ensemble forecasting	Convection-scale weather system forecasts (weather, ~10^0 h)	Wang et al. (2020a); Xu et al. (2022)
	TC forecasts (weather, ~10^0 d)	Zhang et al. (2023a); Zhang et al. (2023b)
	Heatwave forecasts (S2S, ~10^1 d)	Zhang et al. (2024); Zhang et al. (2025)
	Realistic ENSO forecasts (seasonal-to-annual, ~10^2 d)	https://soed.sio.org.cn/emsodm.html; http://cmdp.ncc-cma.net/pred/cn_cmme.php?Elem=CMME-ENSO Duan et al. (2022); Liu et al. (2023b)

method has led to in-depth investigations of the ENSO SPB mechanism and targeted observations for TC forecasting. This endeavor has led to the establishment of our nonlinear theory and technology for significantly reducing prediction errors. This nonlinear theory emphasizes that substantial prediction errors arise from nonlinearities, environmental conditions, and spatial patterns of initial errors, which have gained high recognition from the international atmosphere–ocean community. Nonlinear technology provides a pioneering approach for identifying sensitive areas in targeted observations on the basis of the spatial pattern and geographical location of CNOP and provides a new way to reduce observational and forecast errors via targeted observations through the application of the CNOP method.

Research on high-impact ocean–atmosphere events has further validated this nonlinear theory and technology. In particular, the scientific rigor and effectiveness of this new theory and technology have been verified through its successful application in field campaigns involving targeted observations of TCs and Yellow Sea states conducted by meteorological departments and universities. The CNOP method has been internationally recognized and applied in numerous studies focusing on ensemble forecasts to estimate prediction uncertainties, even in realistic ensemble forecasts. The applications to ensemble forecasts enrich the above nonlinear technology.

With the development of society and the advancement of science and technology, increasing requirements have been placed on numerical weather forecasts and climate predictions for disaster prevention and reduction. These include developments from traditional weather-scale forecasts to multi-scale seamless forecasting and from the independent development of observation networks or numerical models to the coordinated development of observations, data assimilation and numerical models. These requirements present new challenges for predictability research in forecasts/predictions of high-impact weather and climate events. Further studies should focus on developing an efficient and intelligent multi-scale CNOP algorithm for higher-resolution earth system models and applying it to increasingly emerging seamless forecasts and related practical observation experiments and realistic ensemble forecasts in the future. It is also expected that the CNOP method can be applied to address prediction issues of longer timescale variabilities, such as decadal trends and climate shift.

In addition, artificial intelligence (AI) models based on meteorological big data have emerged. These AI models perform well in forecasting specific weather and climate phenomena; however, the data may not be sufficient or may inevitably contain errors; therefore, AI models cannot accurately describe the motions of the atmosphere and ocean. These errors provide new chances for CNOP applications. Recently, pioneering works have been made on AI models for CNOP applications, as benefiting from self-contained optimization modules and related high-efficiency calculations of AI models. Qin et al. (2024) developed a monthly scale AI model for ENSO forecasting and, using the CNOP method, revealed an initial perturbation that has significant effects on ENSO forecasts, while Zu et al. (2025) established a daily scale AI model for forecasts of sea surface temperature in the South China Sea, and identified the time-varying sensitive areas for targeted observation by CNOP. The CNOP applications were also extended to currently widely used AI models. Li et al. (2025)[a] applied the CNOP method to the Fuxi model for determining the sensitive areas of targeted observations associated with forecasts of TCs in the Northwest Pacific, while Zhou et al. (2025)[b] adopted the CNOP to investigate the predictable time of the Pangu-Weather model with respect to Bay of Bengal storm tracks from the perspective of targeted observations. All these studies imply that the combination of the CNOP method and AI models can further advance nonlinear theory and technology for atmospheric and oceanic predictability, vigorously promoting the paradigm shift in the synergistic cognition–observation–model cycle (Mu et al., 2025).

Acknowledgements. The authors greatly appreciate the insightful comments and suggestions of the three anonymous reviewers. This work was sponsored by the National Natural Science Foundation of China (Grant No. 42330111).

REFERENCES

Birgin, E. G., J. M. Martínez, and M. Raydan, 2000: Nonmonotone spectral projected gradient methods on convex sets. *SIAM Journal on Optimization*, **10**(4), 1196–1211, https://doi.org/10.1137/S1052623497330963.

Birgin, E. G., J. M. Martinez, and M. Raydan, 2014: Spectral projected gradient methods: Review and perspectives. *Journal of Statistical Software*, **60**(3), 1–21, https://doi.org/10.18637/jss.v060.i03.

Bishop C. H., Z. Toth, 1999: Ensemble transformation and adaptive observations. *J. Atmos. Sci.*, **56**(11): 1748–1765. https://doi.org/10.1175/1520-0469(1999)056<748:ETAAO>2.0.CO;2.

Buizza, R., and T. N. Palmer, 1995: The singular-vector structure of the atmospheric global circulation. *J. Atmos. Sci*., **52**(9), 1434–1456, https://doi.org/10.1175/1520-0469(1995)052<1434:TSVSOT>2.0.CO;2.

Buizza, R., and A. Montani, 1999: Targeting observations using singular vectors. *J. Atmos. Sci.*, **56**, 2965–2985, https://doi.org/10.1175/1520-0469(1999)056<2965:TOUSV>2.0.CO;2.

Chan, P. W., W. Han, B. Mak, X. H. Qin, Y. Z. Liu, R. Y. Yin, and J. C. Wang, 2023: Ground-space-sky observing system experiment during tropical cyclone Mulan in August 2022. *Adv. Atmos. Sci.*, **40**, 194–200, https://doi.org/10.1007/s00376-022-2267-z.

Chen, B. Y., M. Mu, and X. H. Qin, 2013: The impact of assimilating dropwindsonde data deployed at different sites on Typhoon track forecasts. *Mon. Wea. Rev.*, **141**, 2669–2682, https://doi.org/10.1175/MWR-D-12-00142.1.

Chen, D. K., N. Smith, and W. Kessler, 2018: The evolving ENSO observing system. *National Science Review*, **5**(6), 805–807, https://doi.org/10.1093/nsr/nwy137.

Chen, G., B. Wang, and J. J. Liu, 2021: Study on the sensitivity of initial perturbations to the development of a vortex observed in Southwest China. *J. Geophys. Res.*, **126**, e2021JD034715, https://doi.org/10.1029/2021JD034715.

Chen, Y. Q., D. S. Battisti, T. N. Palmer, J. Barsugli, and E. S. Sarachik, 1997: A study of the predictability of tropical Pacific SST in a coupled atmosphere–ocean model using singular vector analysis: The role of the annual cycle and the ENSO cycle. *Mon. Wea. Rev.*, **125**, 831–845, https://doi.org/10.1175/1520-0493(1997)125<0831:ASOTPO>2.0.CO;2.

Dai, G. K., M. Mu, and L. Wang, 2021: The influence of sudden arctic sea-ice thinning on North Atlantic oscillation events. *Atmos.-Ocean*, **59**(1), 39–52, https://doi.org/10.1080/07055900.2021.1875976.

Ding, R. Q., and J. P. Li, 2007: Nonlinear finite-time Lyapunov exponent and predictability. *Physics Letters A*, **364**, 396–400, https://doi.org/10.1016/j.physleta.2006.11.094.

Duan, Q. Y., F. Pappenberger, A. Wood, H. L. Cloke, and J. C. Schaake, 2019: *Handbook of Hydrometeorological Ensemble Forecasting*. Springer.

Duan, W. S., and M. Mu, 2009: Conditional nonlinear optimal perturbation: Applications to stability, sensitivity, and predictability. *Science in China Series D: Earth Sciences*, **52**(7): 883–906, https://doi.org/10.1007/s11430-009-0090-3.

Duan, W. S., and R. Zhang, 2010: Is model parameter error related to a significant spring predictability barrier for El Nino events? Results from a theoretical model. *Adv. Atmos. Sci.*, **27**, 1003–1013, https://doi.org/10.1007/s00376-009-9166-4.

Duan, W. S., and C. Wei, 2013: The 'spring predictability barrier' for ENSO predictions and its possible mechanism: Results from a fully coupled model. *International Journal of Climatology*, **33**, 1280–1292, https://doi.org/10.1002/joc.3513.

Duan, W. S., and F. F. Zhou, 2013: Non-linear forcing singular vector of a two-dimensional quasi-geostrophic model. *Tellus A*, **65**, 18452, https://doi.org/10.3402/tellusa.v65i0.18452.

Duan, W. S., and Z. H. Huo, 2016: An approach to generating mutually independent initial perturbations for ensemble forecasts: Orthogonal conditional nonlinear optimal perturbations. *J. Atmos. Sci.*, **73**, 997–1014, https://doi.org/10.1175/JAS-D-15-0138.1.

Duan, W. S., and M. Mu, 2018: Predictability of El Niño-southern oscillation events. Oxford Research Encyclopedia of Climate Science, https://doi.org/10.1093/acrefore/9780190228620.013.80.

Duan, W. S., X. C. Liu, K. Y. Zhu, and M. Mu, 2009: Exploring the initial errors that cause a significant "spring predictability barrier" for El Niño events. *J. Geophys. Res.*, **114**, C04022, https://doi.org/10.1029/2008JC004925.

Duan, W. S., X. Q. Li, and B. Tian, 2018: Towards optimal observational array for dealing with challenges of El Niño-Southern Oscillation predictions due to diversities of El Niño. *Climate Dyn.*, **51**, 3351–3368, https://doi.org/10.1007/s00382-018-4082-x.

Duan, W. S., R. Feng, L. C. Yang, and L. Jiang, 2022: A new approach to data assimilation for numerical weather forecasting and climate prediction. *Journal of Applied Analysis & Computation*, **12**(3), 1007–1021, https://doi.org/10.11948/20220098.

Duan, W. S., L. C. Yang, M. Mu, B. Wang, X. S. Shen, Z. Y.

[a] Li, Y., W. Duan, W. Han, and H. Li, 2025: Improving tropical cyclone track forecast skill through assimilating targeted observation achieved by AI-based conditional nonlinear optimal perturbation. (under review)

[b] Zhou, Z. Y., W. Duan, R. Yang, X. H. Qin, and Y. Li, 2025: Uncertainty of "Pangu-weather" in bay of Bengal storm track forecasts: Targeted observation perspective. (under review)

Meng, and R. Q. Ding, 2023a: Recent advances in China on the predictability of weather and climate. *Adv. Atmos. Sci.*, **40**(8), 1521−1547, https://doi.org/10.1007/s00376-023-2334-0.

Duan, W. S., L. C. Yang, Z. Z. Xu, and J. Chen, 2023b: Conditional nonlinear optimal perturbation: Applications to ensemble forecasting of high-impact weather systems. *Numerical Weather Prediction: East Asian Perspectives*, S. K. Park, Ed., Springer, https://doi.org/10.1007/978-3-031-40567-9.

Farrell, B. F., 1990: Small error dynamics and the predictability of atmospheric flows. *J. Atmos. Sci.*, **47**, 2409–2416, https://doi.org/10.1175/1520-0469(1990)047<2409:SEDATP>2.0.CO;2.

Feng, J., X. H. Qin, C. Q. Wu, P. Zhang, L. Yang, X. S. Shen, W. Han, and Y. Z. Liu, 2022: Improving typhoon predictions by assimilating the retrieval of atmospheric temperature profiles from the FengYun-4A's Geostationary Interferometric Infrared Sounder (GIIRS). *Atmospheric Research*, **280**, 106391, https://doi.org/10.1016/j.atmosres.2022.106391.

Gao, Y. J. Y., M. Mu, and G. K. Dai, 2023: Targeted observations on Arctic sea ice concentration for improving extended-range prediction of Ural Blocking. *J. Geophys. Res.*, **128**(22), e2023JD039630, https://doi.org/10.1029/2023JD039630.

Geng, Y., Q. Wang, M. Mu, and K. Zhang, 2020: Predictability and error growth dynamics of the Kuroshio extension state transition process in an eddy-resolving regional ocean model. *Ocean Modelling*, **153**, 101659, https://doi.org/10.1016/j.ocemod.2020.101659.

Han, Z., G. K. Dai, M. Mu, C. X. Li, S. L. Li, X. Y. Ma, and M. B. Zhu, 2023: Extent of the impact of arctic atmospheric uncertainty on extended-range forecasting of cold events in East Asia. *J. Geophys. Res.*, **128**(9), e2022JD037187, https://doi.org/10.1029/2022JD037187.

Harle, M., F. Kwasniok., and U. Feudel, 2006: Growth of finite errors in ensemble prediction. *Nonlinear Processes in Geophysics*, **13**, 167−176, https://doi.org/10.5194/npg-13-167-2006.

Hu, H. Q., J. Y. Liu, L. L. Da, W. H. Guo, K. Liu, and B. L. Cui, 2021: Identification of the sensitive area for targeted observation to improve vertical thermal structure prediction in summer in the Yellow Sea. *Acta Oceanologica Sinica*, **40**(7), 77–87, https://doi.org/10.1007/s13131-021-1738-x.

Huang, J. P., W. Chen, Z. P. Wen, G. J. Zhang, Z. X. Li, Z. Y. Zuo, and Q. Y. Zhao, 2019: Review of Chinese atmospheric science research over the past 70 years: Climate and climate change. *Science China Earth Sciences*, **62**, 1514−1550, https://doi.org/10.1007/s11430-019-9483-5.

Huo, Z. H., and W. S. Duan, 2019: The application of the orthogonal conditional nonlinear optimal perturbations method to typhoon track ensemble forecasts. *Science China Earth Sciences*, **62**, 376−388, https://doi.org/10.1007/s11430-018-9248-9.

Huo, Z. H., W. S. Duan, and F. F. Zhou, 2019: Ensemble forecasts of tropical cyclone track with orthogonal conditional nonlinear optimal perturbations. *Adv. Atmos. Sci.*, **36**(2), 231−247, https://doi.org/10.1007/s00376-018-8001-1.

Jiang, L., W. S. Duan, and H. L. Liu, 2022: The most sensitive initial error of sea surface height anomaly forecasts and its implication for targeted observations of mesoscale eddies. *J. Phys. Oceanogr.*, **52**, 723–740, https://doi.org/10.1175/JPO-D-21-0200.1.

Jiang, L., W. S. Duan, H. Wang, H. L. Liu, and L. J. Tao, 2023: Evaluation of the sensitivity on mesoscale eddy associated with the sea surface height anomaly forecasting in the Kuroshio extension. *Frontiers in Marine Science*, **10**, 1097209, https://doi.org/10.3389/fmars.2023.1097209.

Jiang, L., W. S. Duan, and H. Wang, 2024: The sensitive area for targeting observations of paired mesoscale eddies associated with sea surface height anomaly forecasts. *J. Geophys. Res.*, **129**, e2023JC020572, https://doi.org/10.1029/2023JC020572.

Jiang, Z. N., and M. Mu, 2009: A comparison study of the methods of conditional nonlinear optimal perturbations and singular vectors in ensemble prediction. *Adv. Atmos. Sci.*, **26**(3), 465–470, https://doi.org/10.1007/s00376-009-0465-6.

Ke, J. Y., M. Mu, and X. H. Fang, 2022: Impact of optimally growing initial errors on the mesoscale predictability of heavy precipitation events along the Mei-Yu front in China. *Mon. Wea. Rev.*, **150**, 2399–2421, https://doi.org/10.1175/MWR-D-21-0330.1.

Ke, J. Y., M. Mu, and X. H. Fang, 2023: Influence of physically constrained initial perturbations on the predictability of Mei-Yu heavy precipitation. *Mon. Wea. Rev.*, **151**, 2115–2138, https://doi.org/10.1175/MWR-D-22-0302.1.

Kerswell, R. R., C. C. T. Pringle, and A. P. Willis, 2014: An optimization approach for analysing nonlinear stability with transition to turbulence in fluids as an exemplar. *Reports on Progress in Physics*, **77**, 085901, https://doi.org/10.1088/0034-4885/77/8/085901.

Kramer, W., and H. A. Dijkstra, 2013: Optimal localized observations for advancing beyond the ENSO predictability barrier. *Nonlinear Processes in Geophysics*, **20**, 221–230, https://doi.org/10.5194/npg-20-221-2013.

Levine, A. F. Z., and M. J. McPhaden, 2015: The annual cycle in ENSO growth rate as a cause of the spring predictability barrier. *Geophys. Res. Lett.*, **42**, 5034–5041, https://doi.org/10.1002/2015GL064309.

Li, C. X., G. K. Dai, M. Mu, Z. Han, X. Y. Ma, Z. N. Jiang, J. Y. Zheng, and M. B. Zhu, 2023: Influence of arctic sea-ice concentration on extended-range forecasting of cold events in East Asia. *Adv. Atmos. Sci.*, **40**(12), 2224–2241, https://doi.org/10.1007/s00376-023-3010-0.

Li, Y. N., S. Q. Peng, and D. L. Liu, 2014: Adaptive observation in the South China Sea using CNOP approach based on a 3-D ocean circulation model and its adjoint model. *J. Geophys. Res.*, **119**, 8973–8986, https://doi.org/10.1002/2014JC010220.

Liu, J. Y., K. Liu, W. H. Guo, P. Liang, and L. L. Da, 2023a: Optimal initial errors related to the prediction of the vertical thermal structure and their application to targeted observation: A 3-day hindcast case study in the northern South China Sea. *Deep Sea Research Part I: Oceanographic Research Papers*, **200**, 104146, https://doi.org/10.1016/j.dsr.2023.104146.

Liu, K., W. H. Guo, L. L. Da, J. Y. Liu, H. Q. Hu, and B. L. Cui, 2021: Improving the thermal structure predictions in the Yellow Sea by conducting targeted observations in the CNOP-identified sensitive areas. *Scientific Reports*, **11**, 19518, https://doi.org/10.1038/s41598-021-98994-7.

Liu, T. and Coauthors, 2023b: A multi-model prediction system for ENSO. *Science China Earth Sciences*, **66**, 1231−1240, https://doi.org/10.1007/s11430-022-1094-0.

Lord Rayleigh, F. R. S., 1879: On the stability, or instability, of certain fluid motions. *Proceedings of the London Mathematical Society*, **s1-11**, 57–72, https://doi.org/10.1112/plms/s1-11.1.57.

Lorenz, E. N., 1963: Deterministic nonperiodic flow. *J. Atmos. Sci*, **20**, 130–141, https://doi.org/10.1175/1520-0469(1963)020<0130:DNF>2.0.CO;2.

Lorenz, E. N., 1965: A study of the predictability of a 28-variable atmospheric model. *Tellus*, **17**, 321–333, https://doi.org/10.3402/tellusa.v17i3.9076.

Ma, X. Y., M. Mu, G. K. Dai, Z. Han, C. X. Li, and Z. N. Jiang, 2022: Influence of Arctic sea ice concentration on extended-range prediction of strong and long-lasting Ural blocking events in winter. *J. Geophys. Res.*, **127**, e2021JD036282, https://doi.org/10.1029/2021JD036282.

Magnusson, L., E. Källén, and J. Nycander, 2008: Initial state perturbations in ensemble forecasting. *Nonlinear Processes in Geophysics*, **15**, 751–759, https://doi.org/10.5194/npg-15-751-2008.

Molteni, F., and T. N. Palmer, 1993: Predictability and finite-time instability of the northern winter circulation. *Quart. J. Roy. Meteor. Soc.*, **119**, 269–298, https://doi.org/10.1002/qj.49711951004.

Molteni, F., R. Buizza, T. N. Palmer, and T. Petroliagis, 1996: The ECMWF ensemble prediction system: Methodology and validation. *Quart. J. Roy. Meteor. Soc.*, **122**, 73–119, https://doi.org/10.1002/qj.49712252905.

Moore, A. M., and R. Kleeman, 1996: The dynamics of error growth and predictability in a coupled model of ENSO. *Quart. J. Roy. Meteor. Soc.*, **122**, 1405–1446, https://doi.org/10.1002/qj.49712253409.

Moore, A. M., and R. Kleeman, 1997: The singular vectors of a coupled ocean-atmosphere model of ENSO. I: Thermodynamics, energetics and error growth. *Quart. J. Roy. Meteor. Soc.*, **123**, 953–981, https://doi.org/10.1002/qj.49712354009.

Mu, B., S. C. Wen, S. J. Yuan, and H. Y. Li, 2015a: PPSO: PCA based particle swarm optimization for solving conditional nonlinear optimal perturbation. *Computers & Geosciences*, **83**, 65–71, https://doi.org/10.1016/j.cageo.2015.06.016.

Mu, B., L. L. Zhang, S. J. Yuan, Y. W. Qian, S. C. Wen, J. H. Yan, and F. F. Zhou, 2019: Intelligent algorithms for solving CNOP and their applications in ENSO predictability and tropical cyclone adaptive observations. *Journal of Tropical Meteorology*, **25**(1), 63–81, https://doi.org/10.16555/j.1006-8775.2019.01.007.

Mu, M., 2000: Nonlinear singular vectors and nonlinear singular values. *Science in China Series D: Earth Sciences*, **43**, 375–385, https://doi.org/10.1007/BF02959448.

Mu, M., 2013: Methods, current status, and prospect of targeted observation. *Science China Earth Science*, **56**(12), 1997–2005, https://doi.org/10.1007/s11430-013-4727-x.

Mu, M., and Z. N. Jiang, 2008: A new approach to the generation of initial perturbations for ensemble prediction: Conditional nonlinear optimal perturbation. *Chinese Science Bulletin*, **53**(13), 2062–2068, https://doi.org/10.1007/s11434-008-0272-y.

Mu, M., W. S. Duan, and J. F. Wang, 2002: Nonlinear optimization problems in atmospheric and oceanic sciences. *East-west Journal of Mathematics*, **155**, 169–178.

Mu, M., W. S. Duan, and B. Wang, 2003: Conditional nonlinear optimal perturbation and its applications. *Nonlinear Processes in Geophysics*, **10**, 493–501, https://doi.org/10.5194/npg-10-493-2003.

Mu, M., W. S. Duan, and J. F. Chou, 2004: Recent advances in predictability studies in China (1999-2002). *Adv. Atmos. Sci.*, **21**(3), 437–443, https://doi.org/10.1007/BF02915570.

Mu, M., H. L. Wang, and F. F. Zhou, 2007a: A preliminary application of conditional nonlinear optimal perturbation to adaptive observation. *Chinese Journal of Atmospheric Sciences*, **31**, 1102–1112, https://doi.org/10.3878/j.issn.1006-9895.2007.06.06.

Mu, M., H. Xu, and W. S. Duan, 2007b: A kind of initial errors related to "spring predictability barrier" for El Niño events in Zebiak-Cane model. *Geophys. Res. Lett.*, **34**, L03709, https://doi.org/10.1029/2006GL027412.

Mu, M., W. S. Duan, and B. Wang, 2007c: Season-dependent dynamics of nonlinear optimal error growth and El Niño-Southern Oscillation predictability in a theoretical model. *J. Geophys. Res.*, **112**, D10113, https://doi.org/10.1029/2005JD006981.

Mu, M., F. F. Zhou, and H. L. Wang, 2009: A method for identifying the sensitive areas in targeted Observations for tropical cyclone prediction: Conditional Nonlinear optimal perturbation. *Mon. Wea. Rev.*, **137**, 1623–1639, https://doi.org/10.1175/2008MWR2640.1.

Mu, M., W. S. Duan, Q. Wang, and R. Zhang, 2010: An extension of conditional nonlinear optimal perturbation approach and its applications. *Nonlinear Processes in Geophysics*, **17**, 211–220, https://doi.org/10.5194/npg-17-211-2010.

Mu, M., W. S. Duan, D. K. Chen, and W. D. Yu:, 2015b: Targeted observations for improving initialization of high-impact ocean-atmospheric environmental events forecasting. *National Science Review*, **2**, 226–236, https://doi.org/10.1093/nsr/nwv021.

Mu, M., B. Qin, and G. K. Dai, 2025: Predictability study of weather and climate events related to artificial intelligence models. *Adv. Atmos. Sci.*, **42**(1), 1–8, https://doi.org/10.1007/s00376-024-4372-7.

Mureau, R., F. Molteni, and T. N. Palmer, 1993: Ensemble prediction using dynamically conditioned perturbations. *Quart. J. Roy. Meteor. Soc.*, **119**(510): 299–323, https://doi.org/10.1002/qj.49711951005.

Oosterwijk, A., H. A. Dijkstra, and T. van Leeuwen, 2017: An adjoint-free method to determine conditional nonlinear optimal perturbations. *Computers & Geosciences*, **106**, 190–199, https://doi.org/10.1016/j.cageo.2017.06.014.

Palmer, T. N., R. Gelaro, J. Barkmeijer, and R. Buizza, 1998: Singular vectors, metrics, and adaptive observations. *J. Atmos. Sci.*, **55**, 633–653, https://doi.org/10.1175/1520-0469(1998)055<0633:SVMAAO>2.0.CO;2.

Peng, F., and G. D. Sun, 2014: Application of a derivative-free method with projection skill to solve an optimization problem. *Atmos. Ocean. Sci. Lett.*, **7**(6), 499–504, https://doi.org/10.3878/AOSL20140041.

Pringle, C. C. T., and R. R. Kerswell, 2010: Using nonlinear transient growth to construct the minimal seed for shear flow turbulence. *Physical Review Letters*, **105**(15), 154502, https://doi.org/10.1103/PhysRevLett.105.154502.

Qin, B., Z. Y. Yang, M. Mu, Y. T. Wei, Y. H. Cui, X. H. Fang, G. K. Dai, and S. J. Yuan, 2024: The first kind of predictability problem of El Niño predictions in a multivariate coupled data-driven model. *Quart. J. Roy. Meteor. Soc.*, **150**(765), 5452–5471, https://doi.org/10.1002/qj.4882.

Qin, X. H., W. S. Duan, and M. Mu, 2013: Conditions under which CNOP sensitivity is valid for tropical cyclone adaptive observations. *Quart. J. Roy. Meteor. Soc.*, **139**, 1544–1554, https://doi.org/10.1002/qj.2109.

Qin, X. H., W. S. Duan, P. W. Chan, B. Y. Chen, and K. N. Huang, 2023: Effects of dropsonde data in field campaigns on forecasts of tropical cyclones over the Western North Pacific in 2020 and the role of CNOP sensitivity. *Adv. Atmos. Sci.*, **40**, 791–803, https://doi.org/10.1007/s00376-022-2136-9.

Rivière, O., G. Lapeyre, and O. Talagrand, 2009: A novel technique for nonlinear sensitivity analysis: Application to moist predictability. *Quart. J. Roy. Meteor. Soc.*, **135**, 1520–1537, https://doi.org/10.1002/qj.460.

Samelson, R. M., and E. Tziperman, 2001: Instability of the chaotic ENSO: The growth-phase predictability barrier. *J. Atmos. Sci.*, **58**, 3613–3625, https://doi.org/10.1175/1520-0469(2001)058<3613:IOTCET>2.0.CO;2.

Shen, X. S., J. J. Wang, Z. C. Li, D. H. Chen, and J. D. Gong, 2020: Research and operational development of numerical weather prediction in China. *Journal of Meteorological Research*, **34**(4), 675–698, https://doi.org/10.1007/s13351-020-9847-6.

Shi, B., and G. D. Sun, 2023: An adjoint-free algorithm for conditional nonlinear optimal perturbations (CNOPs) via sampling. *Nonlinear Processes in Geophysics*, **30**, 263–276, https://doi.org/10.5194/npg-30-263-2023.

Snyder, C., 1996: Summary of an informal workshop on adaptive observations and FASTEX. *Bull. Amer. Meteor. Soc.*, **77**, 953–961, https://doi.org/10.1175/1520-0477-77.5.953.

Tang, Y. M., and Coauthors, 2018: Progress in ENSO prediction and predictability study. *National Science Review*, **5**(6), 826–839.

Terwisscha van Scheltinga, A. D., and H. A. Dijkstra, 2008: Conditional nonlinear optimal perturbations of the double-gyre ocean circulation. *Nonlinear Processes in Geophysics*, **15**, 727–734, https://doi.org/10.5194/npg-15-727-2008.

Thompson, P. D., 1957: Uncertainty of initial state as a factor in the predictability of large scale atmospheric flow patterns. *Tellus*, **9**, 275–295, https://doi.org/10.3402/tellusa.v9i3.9111.

Tian, X. J., and X. B. Feng, 2017: A nonlinear least-squares-based ensemble method with a penalty strategy for computing the conditional nonlinear optimal perturbations. *Quart. J. Roy. Meteor. Soc.*, **143**, 641–649, https://doi.org/10.1002/qj.2946.

Toth, Z., and E. Kalnay, 1997: Ensemble forecasting at NCEP and the breeding method. *Mon. Wea. Rev.*, **125**, 3297–3319, https://doi.org/10.1175/1520-0493(1997)125<3297:EFANAT>2.0.CO;2.

Vidard, A., P. A. Bouttier, and F. Vigilant, 2015: NEMOTAM: Tangent and adjoint models for the ocean modelling platform NEMO. *Geoscientific Model Development*, **8**, 1245–1257, https://doi.org/10.5194/gmd-8-1245-2015.

van Oldenborgh, G. J., G. Burgers, S. Venzke, and C. Eckert, 1999: Tracking down the ENSO oscillator with an adjoint OGCM. *Mon. Wea. Rev.*, **127**, 1477–1495, https://doi.org/10.1175/1520-0493(1999)127<1477:TDTEDO>2.0.CO;2.

Wang, B., and X. W. Tan, 2010: Conditional nonlinear optimal perturbations: Adjoint-free calculation method and preliminary test. *Mon. Wea. Rev.*, **138**, 1043–1049, https://doi.org/10.1175/2009MWR3022.1.

Wang, L., X. S. Shen, J. J. Liu, and B. Wang, 2020a: Model uncertainty representation for a convection-allowing ensemble prediction system based on CNOP-P. *Adv. Atmos. Sci.*, **37**(8), 817–831, https://doi.org/10.1007/s00376-020-9262-z.

Wang, Q., and M. Mu, 2015: A new application of conditional nonlinear optimal perturbation approach to boundary condition uncertainty. *J. Geophys. Res.*, **120**, 7979–7996, https://doi.org/10.1002/2015JC011095.

Wang, Q. G., J. F. Chou, and G. L. Feng, 2014: Extracting predictable components and forecasting techniques in extended-range numerical weather prediction. *Science China Earth Sciences*, **57**, 1525–1537, https://doi.org/10.1007/s11430-014-4832-5.

Wang, Q., M. Mu, and G. D. Sun, 2020b: A useful approach to sensitivity and predictability studies in geophysical fluid dynamics: Conditional non-linear optimal perturbation. *National Science Review*, **7**(1), 214–223, https://doi.org/10.1093/nsr/nwz039.

Wang, Q., M. Mu, and S. Pierini, 2020c: The fastest growing initial error in prediction of the Kuroshio extension state transition processes and its growth. *Climate Dyn.*, **54**(3–4), 1953–1971, https://doi.org/10.1007/s00382-019-05097-1.

Webster, P. J., and S. Yang, 1992: Monsoon and ENSO: Selectively interactive systems. *Quart. J. Roy. Meteor. Soc.*, **118**, 877–926, https://doi.org/10.1002/qj.49711850705.

Wei, Y. T., M. Mu, H. L. Ren, and J. X. Fu, 2019: Conditional nonlinear optimal perturbations of moisture triggering primary MJO initiation. *Geophys. Res. Lett*, **46**, 3492–3501, https://doi.org/10.1029/2018GL081755.

Wei, Y. T., H. L. Ren, M. Mu, and J. X. Fu, 2020: Nonlinear optimal moisture perturbations as excitation of primary MJO events in a hybrid coupled climate model. *Climate Dyn.*, **54**, 675–699, https://doi.org/10.1007/s00382-019-05021-7.

WHO, 2011: Targeted observations for improving numerical weather prediction: An overview. WWRP/THORPEX No. 15, 37 pp.

Winkler, J., M. Denhard, and B. A. Schmitt, 2020: Krylov methods for adjoint-free singular vector based perturbations in dynamical systems. *Quart. J. Roy. Meter. Soc.*, **146**, 225–239, https://doi.org/10.1002/qj.3668.

Wu, C. C., J. H. Chen, P. H. Lin, and K. H. Chou, 2007: Targeted observations of tropical cyclone movement based on the adjoint-derived sensitivity steering vector. *J. Atmos. Sci.*, **64**, 2611–2626, https://doi.org/10.1175/JAS3974.1.

Xu, Z. Z., J. Chen, M. Mu, G. K. Dai, and Y. N. Ma, 2022: A nonlinear representation of model uncertainty in a convective-scale ensemble prediction system. *Adv. Atmos. Sci.*, **39**, 1432–1450, https://doi.org/10.1007/s00376-022-1341-x.

Xue, Y., M. A. Cane, and S. E. Zebiak, 1997: Predictability of a coupled model of ENSO using singular vector analysis. Part I: Optimal growth in seasonal background and ENSO cycles. *Mon. Wea. Rev.*, **125**, 2043–2056, https://doi.org/10.1175/1520-0493(1997)125<2043:POACMO>2.0.CO;2.

Yang, L. C., W. S. Duan, Z. F. Wang, and W. Y. Yang, 2022: Toward targeted observations of the meteorological initial state for improving the $PM_{2.5}$ forecast of a heavy haze event that occurred in the Beijing-Tianjin-Hebei region. *Atmospheric Chemistry and Physics*, **22**, 11 429–11 453, https://doi.org/10.5194/acp-22-11429-2022.

Yang, L. C., W. S. Duan, and Z. F. Wang, 2023: An approach to refining the ground meteorological observation stations for improving $PM_{2.5}$ forecasts in the Beijing-Tianjin-Hebei region. *Geoscientific Model Development*, **16**, 3827–3848, https://doi.org/10.5194/gmd-16-3827-2023.

Yu, H. Z., and Z. Y. Meng, 2016: Key synoptic-scale features influencing the high-impact heavy rainfall in Beijing, China, on 21 July 2012. *Tellus A*, **68**, 31045, https://doi.org/10.3402/tellusa.v68.31045.

Yu, H. Z., and Z. Y. Meng, 2022: The impact of moist physics on the sensitive area identification for heavy rainfall associated weather systems. *Adv. Atmos. Sci.*, **39**, 684–696, https://doi.org/10.1007/s00376-021-0278-9.

Yu, Y. S., W. S. Duan, H. Xu, and M. Mu, 2009: Dynamics of nonlinear error growth and season-dependent predictability of El Niño events in the Zebiak–Cane model. *Quart. J. Roy. Meteor. Soc.*, **135**, 2146–2160, https://doi.org/10.1002/qj.526.

Yu, Y. S., M. Mu, and W. S. Duan, 2012: Does model parameter error cause a significant "spring predictability barrier" for El Niño events in the zebiak–cane model. *J. Climate*, **25**, 1263–1277, https://doi.org/10.1175/2011JCLI4022.1.

Yuan, S. J., H. Z. Zhang, Y. X. Liu, and B. Mu, 2022: Feature extraction-based intelligent algorithm framework with neural network for solving conditional nonlinear optimal perturbation. *Soft Computing*, **26**, 6907–6924, https://doi.org/10.1007/s00500-021-06639-8.

Yuan, S. J., Y. X. Liu, B. Qin, B. Mu, and K. Zhang, 2023: A paralleled embedding high-dimensional Bayesian optimization with additive Gaussian kernels for solving CNOP. *Ocean Modelling*, **184**, 102213, https://doi.org/10.1016/j.ocemod.2023.102213.

Zhang, H. Q., and X. J. Tian, 2022: Integral correction of initial and model errors in system of multigrid NLS-4DVar data assimilation for numerical weather prediction (SNAP). *Quart. J. Roy. Meteor. Soc.*, **148**, 2490–2506, https://doi.org/10.1002/qj.4313.

Zhang, H., W. S. Duan, and Y. C. Zhang, 2023a: Using the orthogonal conditional nonlinear optimal perturbations approach to address the uncertainties of tropical cyclone track forecasts generated by the WRF model. *Wea. Forecasting*, **38**, 1907–1933, https://doi.org/10.1175/WAF-D-22-0175.1.

Zhang, L. L., B. Mu, S. J. Yuan, and F. F. Zhou, 2018: A novel approach for solving CNOPs and its application in identifying sensitive regions of tropical cyclone adaptive observations. *Nonlinear Processes in Geophysics*, **25**, 693–712, https://doi.org/10.5194/npg-25-693-2018.

Zhang, L. L., B. Mu, and S. J. Yuan, 2019: A modified direct search algorithm based on Kernel density estimator with three mapping strategies for solving nonlinear optimization. *Journal of Computers*, **30**(4), 17–30, https://doi.org/10.3966/199115992019083004002.

Zhang, Q., G. D. Sun, G. K. Dai, and M. Mu, 2024: Impact of uncertainties in land surface processes on subseasonal predictability of heat waves onset over the Yangtze River Valley. *J. Geophys. Res.*, **129**, e2023JD038674, https://doi.org/10.1029/2023JD038674.

Zhang, Q. Y., M. Mu, and G. D Sun, 2025: Skillful subseasonal ensemble predictions of heat wave onsets through better representation of land surface uncertainties. *npj Climate and Atmospheric Science*, **8**, 15, https://doi.org/10.1038/s41612-024-00876-y.

Zhang, Y. C., W. S. Duan, S. Vannitsem, and H. Zhang, 2023b: A new approach to represent model uncertainty in the forecasting of tropical cyclones: The orthogonal nonlinear forcing singular vectors. *Quart. J. Roy. Meteor. Soc.*, **149**(755), 2206–2232, https://doi.org/10.1002/qj.4502.

Zhao, H. D., Y. X. Zhao, and X. Deng, 2023: Application of the most sensitive initial error in sampling scheme design for a mobile platform. *Journal of Marine Science and Engineering*, **11**(11), 2080, https://doi.org/10.3390/jmse11112080.

Zu, Z. Q., and Coauthors, 2025: How do deep learning forecasting models perform for surface variables in the South China Sea compared to operational oceanography forecasting systems? *Adv. Atmos. Sci.*, **42**(1), 178–189, https://doi.org/10.1007/s00376-024-3264-1.