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Citation: [SCIENCE CHINA Earth Sciences](#) ; doi: 10.1007/s11430-018-9248-9

View online: <http://engine.scichina.com/doi/10.1007/s11430-018-9248-9>

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The application of the orthogonal conditional nonlinear optimal perturbations method to typhoon track ensemble forecasts

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Received January 24, 2018; revised April 3, 2018; accepted July 19, 2018; published online September 4, 2018

Abstract The orthogonal conditional nonlinear optimal perturbations (CNOPs) method, orthogonal singular vectors (SVs) method and CNOP+SVs method, which is similar to the orthogonal SVs method but replaces the leading SV (LSV) with the first CNOP, are adopted in both the Lorenz-96 model and Pennsylvania State University/National Center for Atmospheric Research (PSU/NCAR) Fifth-Generation Mesoscale Model (MM5) for ensemble forecasts. Using the MM5, typhoon track ensemble forecasting experiments are conducted for strong Typhoon Matsa in 2005. The results of the Lorenz-96 model show that the CNOP+SVs method has a higher ensemble forecast skill than the orthogonal SVs method, but ensemble forecasts using the orthogonal CNOPs method have the highest forecast skill. The results from the MM5 show that orthogonal CNOPs have a wider horizontal distribution and better describe the forecast uncertainties compared with SVs. When generating the ensemble mean forecast, equally averaging the ensemble members in addition to the anomalously perturbed forecast members may contribute to a higher forecast skill than equally averaging all of the ensemble members. Furthermore, for given initial perturbation amplitudes, the CNOP+SVs method may not have an ensemble forecast skill greater than that of the orthogonal SVs method, but the orthogonal CNOPs method is likely to have the highest forecast skill. Compared with SVs, orthogonal CNOPs fully consider the influence of nonlinear physical processes on the forecast results; therefore, considering the influence of nonlinearity may be important when generating fast-growing initial ensemble perturbations. All of the results show that the orthogonal CNOP method may be a potential new approach for ensemble forecasting.

Keywords Ensemble forecasts, Initial perturbation, Conditional nonlinear optimal perturbation, Singular vector, Typhoon track

Citation: Huo Z, Duan W. 2018. The application of the orthogonal conditional nonlinear optimal perturbations method to typhoon track ensemble forecasts. *Science China Earth Sciences*, 61, <https://doi.org/10.1007/s11430-018-9248-9>

1. Introduction

Due to the chaotic characteristics of the atmospheric system and the existence of initial errors and model errors, a single and deterministic forecast is simply an estimation of the future atmospheric state and, thus, has uncertainties. To reduce forecast error, Epstein (1969) suggested the explicit integration of the Liouville equations to obtain probability

distribution information of the atmospheric state. However, the degree of freedom for a numerical weather forecast system reaches the order of millions, which indicates that the integration of the Liouville equations is impractical due to huge computational costs. Subsequently, Leith (1974) proposed the Monte Carlo forecasting (MCF) approach, which adds random perturbations to the initial analysis field to generate a set of forecast members and estimate the probability density function (PDF) of the forecast state. This is the basic idea behind ensemble forecasting. Epstein and

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Leith hypothesized that if a set of initial perturbations could well describe the uncertainty of the initial condition, then the forecasts obtained by integrating the perturbed initial conditions could be used to estimate the uncertainty of the real atmospheric state. Generally, the ensemble mean filters the unpredictable components of different forecast members and maintains the common predictable components of the forecast members. Therefore, the ensemble mean could improve a single and deterministic forecast (Leith, 1974; Leutbecher and Palmer, 2008). Furthermore, ensemble forecasts could provide probability information for events occurring compared with the single and deterministic forecast and, thus, could serve society more adequately.

In fact, there are several ensemble forecasting methods used in operational forecasts. For example, the linear singular vectors (SVs) method (Lorenz, 1965; Molteni et al., 1996; Mureau et al., 1993) has been successfully applied at the European Centre for Medium Range Weather Forecasts (ECMWF). However, SVs are a set of orthogonal initial perturbations that have maximum linear growth rates in different subspaces, and they do not consider the influence of nonlinear physical processes on the forecast results. Gilmour and Smith (1997) noted that the SVs method has linear limitations in the generation of ensemble initial perturbations. Anderson (1997) indicated that SVs are only sensitive to the evolution of initial perturbations in the linear regime and could not describe extreme perturbations. Barkmeijer et al. (2001) noted that SVs with large and spurious perturbation growth in the upper troposphere may arise when computing tropical SVs. Li et al. (2005) showed that the Florida State University Global Spectral Model (FSUGSM) appears to have more leading SVs dominated by spurious modes than the ECMWF model. Reynolds et al. (2009) found that the linear calculations lead to SVs that have case-by-case variability when reflecting actual nonlinear perturbation growth. Hence, the SVs method has linear limitations in the generation of initial perturbations for ensemble forecasts.

To consider the influence of nonlinearity, Mu et al. (2003) proposed the conditional nonlinear optimal perturbation (CNOP) approach. As a nonlinear extension of leading SV (LSV), the CNOP is an initial perturbation that satisfies the physical constraints and has the largest nonlinear evolution at the time of the forecast (Duan et al., 2004; Mu and Zhang, 2006; Duan and Mu, 2009). Considering the linear limitation of LSV, Mu and Jiang (2008) replaced the LSV with the CNOP and maintained the other SVs (hereafter called the CNOP+SVs method for simplicity) to conduct ensemble forecasting experiments; this study achieved higher forecast skill than those using the orthogonal SVs method. The results showed that considering the influence of nonlinearity when generating fast-growing initial perturbations is important. However, this study simply replaced LSV with CNOP, and other SVs still had linear limitations.

To guarantee the diversity of the ensemble members and fully consider the influence of nonlinear physical processes on the forecast results, Duan and Huo (2016) proposed the orthogonal CNOPs method and applied it to the Lorenz-96 model. The results showed that ensemble forecasts using the orthogonal CNOPs method could contribute to higher forecast skill compared with the orthogonal SVs method. However, Duan and Huo (2016) did not compare the ensemble forecast skills of the orthogonal CNOPs method and CNOP+SVs method. Since the CNOP+SVs method may also contribute to higher ensemble forecast skill than the orthogonal SVs method (Mu and Jiang, 2008; Jiang and Mu, 2009), does the orthogonal CNOPs method perform better than the CNOP+SVs method? In addition, Duan and Huo (2016) note that when the initial analysis errors grow rapidly, the orthogonal CNOPs method may have a higher ensemble forecast skill; they deduce that the orthogonal CNOPs method may have a higher ensemble forecast skill for strong weather and climate events. In fact, a typhoon is a strong event that brings severe disastrous effects to the whole world and leads to huge losses in national economy, lives and property. Therefore, for strong typhoon events, do ensemble forecasts using the orthogonal CNOPs method contribute to higher ensemble forecast skill than those using the orthogonal SVs method or CNOP+SVs method?

To answer these two questions, this paper first compares the ensemble forecast skills of the orthogonal SVs method, orthogonal CNOPs method and CNOP+SVs method with that of the Lorenz-96 model (Lorenz, 1996). Then, to test the effectiveness of the orthogonal CNOPs method for typhoon ensemble forecasts, the Pennsylvania State University/National Center for Atmospheric Research (PSU/NCAR) Fifth-Generation Mesoscale Model (MM5) (MM5; Dudhia, 1993) is adopted in this paper.

2. Experimental strategy

2.1 Models and Typhoon Matsa

In this paper, we use the Lorenz-96 model to compare the ensemble forecast skills of the orthogonal CNOPs method, orthogonal SVs method and CNOP+SVs method. The Lorenz-96 model has been used to study various questions associated with predictability, especially in the field of ensemble forecasting (Roulston and Smith, 2003; Descamps and Talagrand, 2007; Revelli et al., 2010; Basnarkov and Kocarev, 2012; Li et al., 2013; Feng et al., 2014; Ding et al., 2017). The Lorenz-96 model is governed by the following differential equation with cyclic boundary conditions:

$$dX_j / dt = (X_{j+1} - X_{j-2})X_{j-1} - X_j + F, \quad (1)$$

where $j=1, \dots, m$, $m=40$ represents the number of variables, X_j represents the variable being analyzed, and $F=8$ is the

external forcing value. For details on the Lorenz-96 model, please refer to [Duan and Huo \(2016\)](#).

To test the usefulness of the orthogonal CNOPs method in typhoon ensemble forecasts, we adopt the MM5 and its corresponding tangent linear model and adjoint model ([Zou et al., 1997](#)) to conduct the ensemble forecasting experiments. The MM5 is a mesoscale model developed by the National Center for Atmospheric Research (NCAR) and Pennsylvania State University (PSU). The MM5 possesses effective tangent-linear and adjoint models ([Zou et al., 1997](#)), which makes the computation of SVs and CNOPs possible. Furthermore, the MM5 and its tangent-linear and adjoint models have been widely adopted in studies on the predictability of extreme weather events, such as typhoons and heavy rain ([Cheung, 2001](#); [Hao et al., 2007](#); [Wang and Liang, 2007](#); [Mu et al., 2009](#); [Zhou and Mu, 2011](#); [Qin et al., 2013](#); [Mu et al., 2014](#); [Yu et al., 2017](#)). Their results show that the MM5 is an alternative platform for examining the usefulness of new approaches. Therefore, it is suitable for us to examine the usefulness of orthogonal CNOPs in yielding initial perturbations for ensemble forecasts by using the MM5. In our numerical experiments, physical parameterizations include the Anthes-Kuo cumulus parameterization scheme, high-resolution PBL scheme, simple cooling radiation scheme, and stable precipitation scheme.

In this study, the horizontal resolution of the model was 60 km, with a model domain of 51×61 grids (y -direction $\times$ x -direction), and the vertical direction was evenly divided into 20 σ levels. The initial and boundary conditions were supplied by the FNL (Final) Operational Global Analysis ($1^\circ \times 1^\circ$) from the National Centers for Environmental Prediction (NCEP), which were interpolated onto the MM5 grids. By integrating the MM5 with the initial and boundary conditions, we obtained a control forecast. Historical tropical cyclone data (available online at http://tcdata.typhoon.org.cn/en/zjljsjj_zlhq.html; [Ying et al., 2014](#)) supplied by the Tropical Cyclone Data Center of the China Meteorological Administration were used to evaluate the forecast results.

Here, we randomly chose STY Matsa in 2005 to study the ensemble forecast skills of different methods. Matsa made landfall in China and brought severe and disastrous effects to China. For STY Matsa in 2005, the model domain center is 28°N , 122°E , and the 5-day forecast period is from 12:00 on August 3, 2005, to 12:00 on August 8, 2005 (UTC), which starts 2 days before the landing of Matsa in the Zhejiang Province and contains the processes of landfall and propagation during Matsa. It is noted that we randomly choose the initial forecast time so that the whole forecast period covers the processes before, during and after the tropical cyclone made landfall in China. In addition, the domain is randomly selected to encompass the tropical cyclone track during the forecast period.

2.2 Initial perturbation schemes

For the Lorenz-96 model, based on [Duan and Huo \(2016\)](#), we further replace the LSV with the first CNOP and maintain the other SVs to obtain 15 initial perturbations. Then, these 15 initial perturbations are superimposed on and subtracted from the initial analysis field to obtain 30 perturbed initial conditions. Combined with the unperturbed initial analysis field, there are 31 initial conditions. By integrating the Lorenz-96 model with these 31 initial conditions, we can obtain 31 ensemble members. Then, the ensemble forecast skills of different methods are compared. For details on the computation of orthogonal CNOPs and SVs, please refer to [Duan and Huo \(2016\)](#).

To evaluate the ensemble forecast skills of the orthogonal CNOPs method for typhoon forecasts, we adopt the orthogonal CNOPs method, orthogonal SVs method, and CNOP+SVs method to generate the ensemble members and compare the ensemble forecast skills of these methods. Considering the limitations of the computational resources, we adopt each of these methods to generate 5 initial perturbations in this study, and then add them to or subtract them from the initial analysis field to obtain 10 perturbed initial fields. By integrating the initial analysis field, we can obtain the control forecast. Integrating the 10 perturbed initial fields gives us the other 10 ensemble members. Thus, we can obtain 11 ensemble members in total for each method. The initial perturbation scheme for each method is described as follows.

The computation of orthogonal CNOPs is the same as that in [Duan and Huo \(2016\)](#). It is noted that two norms are related to compute orthogonal CNOPs. One norm is used to measure the amplitude of the initial perturbations, and the other norm is used to evaluate the amplitude of the nonlinear evolution of initial perturbations. The fact that the dry total energy norm accounts for wind, temperature and surface-pressure disturbances makes it a suitable candidate for these two norms. Several studies have pointed out that the most appropriate initial time norm in an SVs-based ensemble prediction system should be based on the inverse of the analysis-error covariance matrix ([Ehrendorfer and Tribbia, 1997](#); [Palmer et al., 1998](#)). However, the spectrum of dominant SVs with respect to the dry total energy norm is consistent with the spectrum of estimates for the analysis-error variance ([Palmer et al., 1998](#)). Therefore, among the simple norms, the total dry energy norm is a reasonable first-order approximation of the analysis-error covariance metric ([Buizza et al., 1997](#)), and the total dry energy SVs are probably reasonable substitutes for the analysis-error covariance SVs ([Molteni et al., 1996](#)). As a result, the most commonly used norm at both the initial and final times in the ensemble prediction system is the dry total energy norm. Specifically, the dry total energy norm has been widely

adopted in studies on atmospheric predictability and targeted observations using the CNOP and SV methods based on the MM5 and its tangent linear and adjoint models (Mu et al., 2009; Zhou and Mu, 2011; Qin et al., 2013; Mu et al., 2014; Yu et al., 2017). In this study, for all initial perturbation methods, we perturbed the zonal wind, meridional wind, temperature and surface pressure and used the dry total energy norm for each of these two norms. The main conclusions of this paper are anticipated to be unchanged when the norms are changed since the main differences between the orthogonal CNOPs and SVs are thought to be caused by nonlinear processes.

Specifically, the dry total energy norm is described as follows:

$$\|\delta \mathbf{X}\|^2 = \delta \mathbf{X}^T \mathbf{C} \delta \mathbf{X} = \frac{1}{D} \int_D \int_0^1 \left[\mathbf{u}'^2 + \mathbf{v}'^2 + \frac{c_p}{T_r} \mathbf{T}'^2 + R_a T_r \left(\frac{\mathbf{P}_s'}{P_r} \right)^2 \right] d\sigma dD, \quad (2)$$

where the perturbation $\delta \mathbf{X}$ is composed of $\mathbf{u}'$, $\mathbf{v}'$, $\mathbf{T}'$ and $\mathbf{P}_s'$, which represent the perturbed zonal and meridional winds, temperature, and surface pressure components, respectively; $\mathbf{C}$ represents the operator corresponding to the norm; D represents the horizontal domain of integration; σ represents the vertical coordinate; $c_p = 1005.71 \text{ J kg}^{-1} \text{ K}^{-1}$ represents the specific heat at a constant pressure; $R_a = 287.04 \text{ J kg}^{-1} \text{ K}^{-1}$ is the gas constant of dry air; and $P_r = 1000 \text{ hPa}$, $T_r = 270 \text{ K}$ are reference parameters. Specifically, the cost function to compute orthogonal CNOPs has the following equation:

$$J(\delta \mathbf{X}_{0,j}^*) = \max_{\delta \mathbf{X}_0 \in \Omega_j} [M(\mathbf{X}_0 + \delta \mathbf{X}_0) - M(\mathbf{X}_0)]^T \times \mathbf{C}_2 [M(\mathbf{X}_0 + \delta \mathbf{X}_0) - M(\mathbf{X}_0)], \quad (3)$$

$$\Omega_j = \begin{cases} \left\{ \delta \mathbf{X}_{0,j} \in \mathbb{R}^n \mid \delta \mathbf{X}_{0,j}^T \mathbf{C}_1 \delta \mathbf{X}_{0,j} \leq \beta \right\}, j = 1, \\ \left\{ \delta \mathbf{X}_{0,j} \in \mathbb{R}^n \mid \delta \mathbf{X}_{0,j}^T \mathbf{C}_1 \delta \mathbf{X}_{0,j} \leq \beta, \right. \\ \left. \delta \mathbf{X}_{0,j} \perp \Omega_k, k = 1, \dots, j-1 \right\}, j > 1, \end{cases} \quad (4)$$

where M is the propagator of the nonlinear model, $\mathbb{R}$ denotes the set of real numbers, n is the dimension of the vector space, $\delta \mathbf{X}_0^T \mathbf{C}_1 \delta \mathbf{X}_0 \leq \beta$ represents the constraint condition, $\beta > 0$ is a positive constant with a unit of J kg^{-1} , and $\perp$ represents the orthogonality among different vectors. Here, the initial perturbations focus on the whole model domain (i.e., the horizontal domain for the integration of D_1 corresponding to $\mathbf{C}_1$ represents the whole model domain, D_1). However, computational instability usually occurs while computing orthogonal CNOPs when the horizontal domain of integration (D_2) corresponding to $\mathbf{C}_2$ represents the whole model domain. Hence, we cut off three grids near the boundary of the horizontal model domain to achieve a remaining simulation model domain of size D_2 . Similar to Duan and Huo (2016), we first compute the global CNOP (i.e., the first

CNOP) and then compute the second CNOP in the initial perturbation subspace orthogonal to the first CNOP. Second, we compute the third CNOP in the initial perturbation subspace orthogonal to the first CNOP and the second CNOP. By repeating this process, we can obtain the first 5 orthogonal CNOPs.

It is noted that different methods are adopted in this study under the same conditions. The phrase “under the same condition” means that the perturbation variables, horizontal model domain for the initial perturbations (i.e., D_1), norm to measure the perturbations, and optimization time (for the orthogonal CNOPs and orthogonal SVs methods) are consistent for different methods. This phrase is adopted here to exclude interference due to the nonconformity of conditions, which allows for the comparison of ensemble forecast skills from different methods to be relatively fair. Next, the processes of generating the initial perturbations for the orthogonal SVs method and CNOP+SVs method are introduced sequentially.

Based on the cost function for computing orthogonal CNOPs, the cost function for computing SVs follows the eq. (5).

$$J(\delta \mathbf{X}_0^*) = \max_{\delta \mathbf{X}_0 \in \Omega} \frac{[L\delta \mathbf{X}_0]^T \mathbf{C}_2 [L\delta \mathbf{X}_0]}{\delta \mathbf{X}_0^T \mathbf{C}_1 \delta \mathbf{X}_0}, \quad (5)$$

where L is the propagator of the linearized model, $\Omega = \left\{ \delta \mathbf{X}_0 \in \mathbb{R}^n \mid \delta \mathbf{X}_0^T \mathbf{C}_1 \delta \mathbf{X}_0 \leq \beta \right\}$ represents the constraint condition of the initial perturbations. SVs cause the cost function to obtain maxima for the initial perturbations in orthogonal subspaces. According to eq. (5), we can compute the first 5 orthogonal SVs. It is noted that the ensemble forecast skills for different methods are compared under the same conditions. Therefore, we scale the first 5 SVs so that they have the same amplitude as that for the orthogonal CNOPs. Then, the scaled SVs are used to generate the ensemble members.

When ensemble forecasts are conducted with the CNOP +SVs method, we replace the scaled LSV with the first CNOP and maintain the other scaled SVs. Specifically, we take the first CNOP and the remaining scaled SVs as the initial perturbations, and then add them to and subtract them from the initial analysis field to conduct the ensemble forecasting experiments.

2.3 Evaluation

After obtaining 11 ensemble members, we evaluate the ensemble forecast skills. Here, we mainly evaluate the ensemble forecast skills of different methods by computing the track forecast error of the ensemble mean. The tropical cyclone center is determined by the location of minimum sea-level pressure. Assuming that $\mathbf{F}_i$ represents the forecast track

of the i -th ensemble member, and N represents the ensemble size, then the ensemble mean track $\mathbf{F}$ is determined by the following equation:

$$\mathbf{F} = \frac{1}{N} \sum_{i=1}^N \mathbf{F}_i \quad (6)$$

Here, the forecast error of the tropical cyclone center location is determined by the great-circle distance between two points on Earth. Assuming that $\mathbf{F}=(x_f, y_f)$ represents the forecast location, $\mathbf{A}=(x_o, y_o)$ represents the observation location, x_f and x_o represent the longitudinal coordinates, and y_f and y_o represent the latitudinal coordinates, then the forecast error of a tropical cyclone center location is expressed as:

$$|\mathbf{F} - \mathbf{A}| = 111.11 \cdot \cos^{-1} \times [\sin(y_o)\sin(y_f) + \cos(y_o)\cos(y_f)\cos(x_o - x_f)] \quad (7)$$

Based on eq. (7), the forecast error of the ensemble mean track is expressed as $e = |\mathbf{F} - \mathbf{A}|$. To measure the forecast skill of the ensemble mean, we define an improvement of the ensemble mean to the control forecast as s :

$$s = \frac{E_c - E_e}{E_c} \times 100\%, \quad (8)$$

where E_c represents the forecast error of the control forecast and E_e represents the forecast error of the ensemble mean. The ensemble mean improves the control forecast when $s > 0$. Otherwise, the ensemble mean does not improve the control

forecast and is actually worse than that of the control forecast. The larger s is, the higher the forecast skill of the ensemble mean.

3. Ensemble forecast results with the Lorenz-96 model

Based on Duan and Huo (2016), the ensemble forecast results with the Lorenz-96 model show that the orthogonal CNOPs method has a higher forecast skill than the orthogonal SVs method. However, Mu and Jiang (2008) show that the CNOP+SVs method may also have a higher ensemble forecast skill than the SVs method. Therefore, for ensemble forecasts with the Lorenz-96 model, whether the CNOP+SVs method behaves better than the orthogonal SVs method is unknown. In addition, whether the orthogonal CNOPs method has a higher ensemble forecast skill than the CNOP+SVs method is also unknown. To solve these problems, we apply the CNOP+SVs method in the ensemble forecasts with the Lorenz-96 model and compare the ensemble forecast skills of the orthogonal CNOPs method, orthogonal SVs method and CNOP+SVs method (Figure 1). The results show that the CNOP+SVs method has a higher forecast skill than the orthogonal SVs method, but the orthogonal CNOPs method has the best ensemble forecast

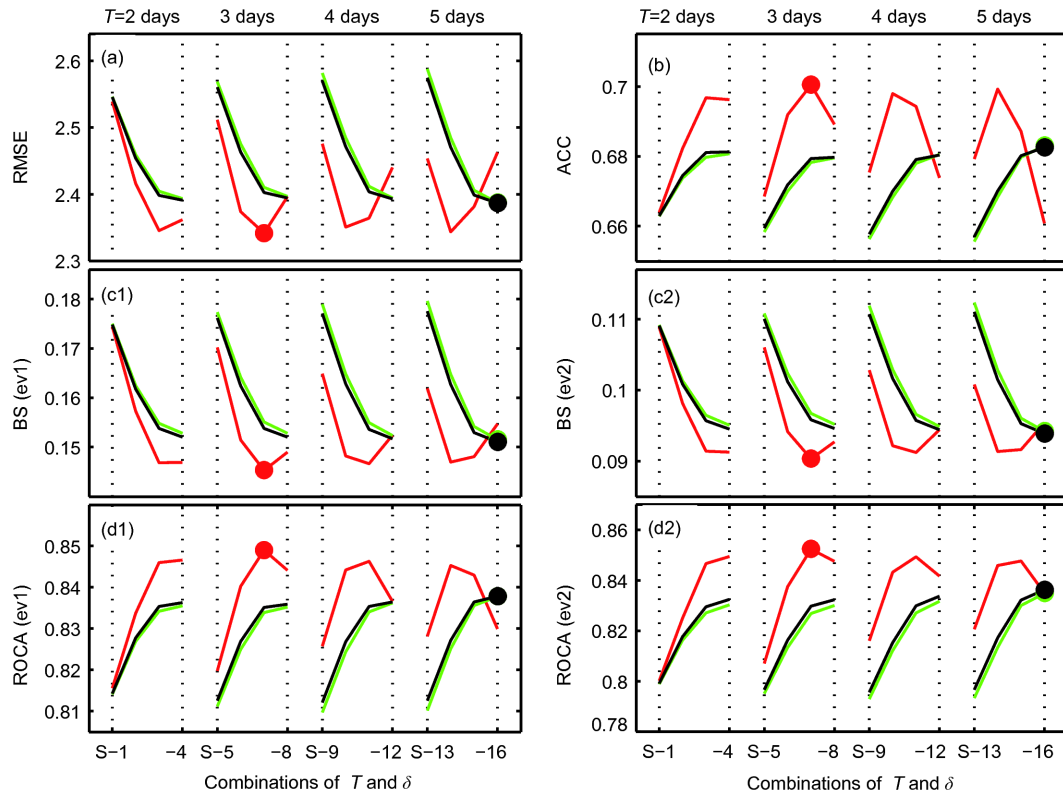


Figure 1 Ensemble forecast results for different methods with the Lorenz-96 model: the orthogonal CNOPs method (red lines), orthogonal SVs method (green lines), and CNOP+SVs method (black lines). The dots denote the highest forecast skill for each method. The horizontal axis denotes different schemes used to compute the initial perturbations, and the vertical axis denotes different evaluation scores. For details, please refer to Duan and Huo (2016).

skill. This means that only replacing the LSV with the first CNOP (but maintaining the other SVs) still has linear limitations, and fully considering the influence of nonlinearity is important when generating fast-growing initial perturbations for ensemble forecasts.

4. Ensemble forecast results for STY Matsa in 2005

To test the effectiveness of the orthogonal CNOPs method in this study, we study the typhoon track ensemble forecast skills of the orthogonal CNOPs method for the case of STY Matsa in 2005. Figure 2a gives the observed typhoon tracks (red line) and the control forecast (black line). Figure 2b gives the evolution of the track forecast error for the control forecast compared with the observed track. The results show that the forecast error of the control forecast increases quickly, and the forecast error of the control forecast at a lead time of 24 h is 113.4 km, which is reasonable (Chou et al., 2011; Yu et al., 2012). This means that experiments with the MM5 are feasible.

Compared with the orthogonal SVs, orthogonal CNOPs consider the influence of nonlinear physical processes on forecast results. Hence, the spatial structures of orthogonal SVs and orthogonal CNOPs should have large differences. In Section 4.1, we compare the spatial structures of orthogonal SVs and orthogonal CNOPs. Because the ensemble forecast skills are closely related to the amplitude of the initial perturbations, we specifically analyze the influence of the amplitude of the initial perturbations on the ensemble forecast skills of the orthogonal CNOPs method and discuss how to generate an ensemble mean with a higher forecast skill in Section 4.2. Finally, we compare the ensemble forecast skills of the orthogonal CNOPs method, orthogonal SVs method and CNOP+SVs method in Section 4.3.

4.1 The spatial structures of orthogonal SVs and orthogonal CNOPs

Previous studies have shown that there is a great difference between CNOP and LSV. According to the cost functions used to compute orthogonal SVs and CNOPs, orthogonal CNOPs and SVs may have large differences when nonlinearity is strong. Therefore, we set $\beta=0.3$ to compute orthogonal CNOPs and SVs and compare their spatial structures. The results are shown in Figure 3, which shows that orthogonal CNOPs have a wider spatial distribution and for each CNOP, there is a vortex center that is located near the center of STY Matsa at the initial time (as shown in Figure 2a). However, among the first 5 SVs, only SV 3 and SV 4 present a vortex around the typhoon center. This indicates that orthogonal CNOPs better describe the initial uncertainty of typhoons and may have higher forecast skill. Furthermore, the spatial structures of orthogonal SVs and orthogonal CNOPs are largely different, which reflects the large influence of nonlinear physical processes on forecast results. In addition to the first CNOP and LSV, there are also great differences between the other CNOPs and their corresponding SVs. These differences sufficiently indicate that considering the influence of nonlinearity when generating fast-growing initial perturbations may be necessary.

4.2 Impact of the amplitudes of initial perturbations on ensemble forecast skills using the orthogonal CNOPs method

In this section, we separately set $\beta=0.3$, $\beta=0.3 \times 4$, and $\beta=0.3 \times 9$ to compute orthogonal CNOPs for the ensemble forecasts. For simplicity, the ensemble mean for the orthogonal CNOPs method is called the CNOPs-ensemble mean. First, we define the ensemble mean as the equally weighted mean by averaging over all ensemble members and compare

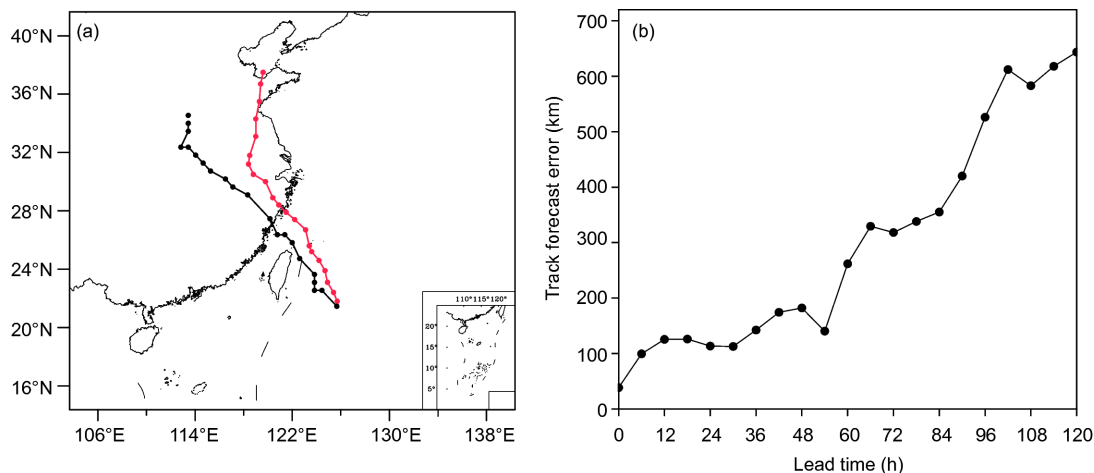


Figure 2 The control forecast results of STY Matsa in 2005. (a) The observed track (red line) and the control forecast track (black line); (b) the evolution of the forecast error for the control forecast track relative to the observed track.

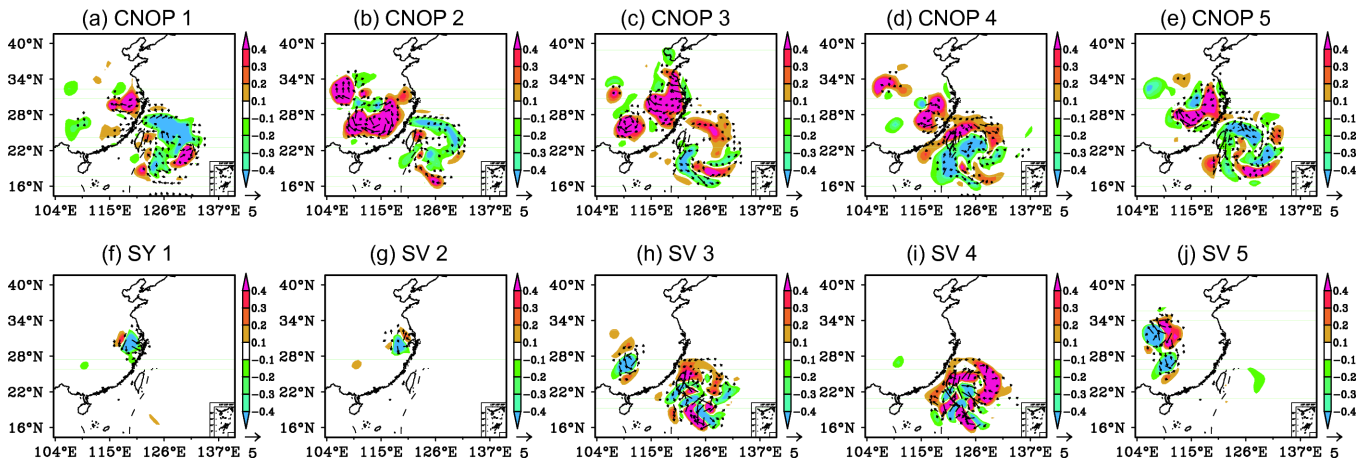


Figure 3 The spatial structures of the temperature component (shaded) and wind component (vector) for the first 5 orthogonal CNOPs ((a)–(e)) and SVs ((f)–(j)) at model level $\sigma=0.975$ for $\beta=0.3$.

the corresponding forecast skills for different amplitudes of the initial perturbations. Figure 4a shows the control forecast track, observed typhoon track and forecast tracks of the equally weighted CNOPs-ensemble mean with different amplitudes of β . The results show that the forecast track of the CNOPs-ensemble mean is closer to the observed track as β increases. The CNOPs-ensemble mean successfully forecasts the landfall location of Matsa and resembles the observed track the best when $\beta=0.3\times 9$. Figure 4b gives the evolution of the track forecast error for the control forecast track and equally weighted CNOPs-ensemble mean with different amplitudes of β . Table 1 gives the corresponding average track forecast error over a 5-day forecast period. The results show that the average track forecast error of the CNOPs-ensemble mean is smaller than that of the control forecast, but the extent is small. Specifically, the CNOPs-ensemble mean has a comparable track forecast error when $\beta=0.3\times 4$ and $\beta=0.3\times 9$. This is not consistent with the CNOPs-ensemble mean tracks shown in Figure 4a. By analyzing the forecasts at different forecast times, we find that the typhoon center of the CNOP-ensemble mean has a forward speed that is too fast when $\beta=0.3\times 9$. Hence, when $\beta=0.3\times 9$ (though the CNOPs-ensemble mean is closest to the observed track regarding the forecast track), the track forecast error of the CNOPs-ensemble mean is not the smallest.

The above results correspond to the equally weighted CNOPs-ensemble mean. In fact, Toth et al. (2001) showed that in a reliable ensemble prediction system, most ensemble members resemble the observations, and high forecast skills usually correspond to the fact that most ensemble members have relatively consistent results. Therefore, increasing the weights of ensemble members whose forecast results are relatively consistent usually has a positive impact on the ensemble mean (Zhang and Krishnamurti, 1997; Duan and Wang, 2006; Elsberry et al., 2008). However, what does this indicate about the distribution of ensemble members ob-

tained with the orthogonal CNOPs method? Are there anomalously perturbed forecast members? To answer these questions, we analyze the tracks of ensemble members corresponding to orthogonal CNOPs with different amplitudes and the track errors of ensemble members relative to the control forecast. Figure 5a and 5b respectively give the tracks of the ensemble members and the evolutions of the track errors for the ensemble members relative to the control forecast for $\beta=0.3\times 9$. Figure 5c and 5d give the same results but for $\beta=0.3\times 4$, and Figures 5e and 5f give the same results but for $\beta=0.3$. The results show that there exists an anomalously perturbed forecast member whether $\beta=0.3$, $\beta=0.3\times 4$ or $\beta=0.3\times 9$. The tracks of the anomalously perturbed forecast members are shown with purple lines in Figure 5a, 5c and 5e, and the track errors of the anomalously perturbed forecast members relative to the control forecast are shown with purple lines in Figure 5b, 5d and 5f. The results show that the anomalously perturbed forecast has much larger relative track errors in the early period than those from other perturbed forecast members.

Here, to remove the negative impact of the anomalously perturbed forecast on the ensemble mean, when there is an anomalously perturbed forecast, we compute the ensemble mean as the equally weighted ensemble mean by averaging the ensemble members in addition to the anomalously perturbed forecasts. If there is not an anomalously perturbed forecast, we compute the ensemble mean with all ensemble members. Figure 6a gives the tracks of the control forecast, observations, and CNOPs-ensemble means for the initial perturbations with different amplitudes. Figure 6b gives the evolution of the track forecast errors for the control forecast and CNOPs-ensemble means. The results show that the track of the CNOPs-ensemble mean better resembles the observations when the initial perturbations are larger, and the CNOPs-ensemble mean successfully forecasts the landfall location of the typhoon when $\beta=0.3\times 9$. In addition, the track

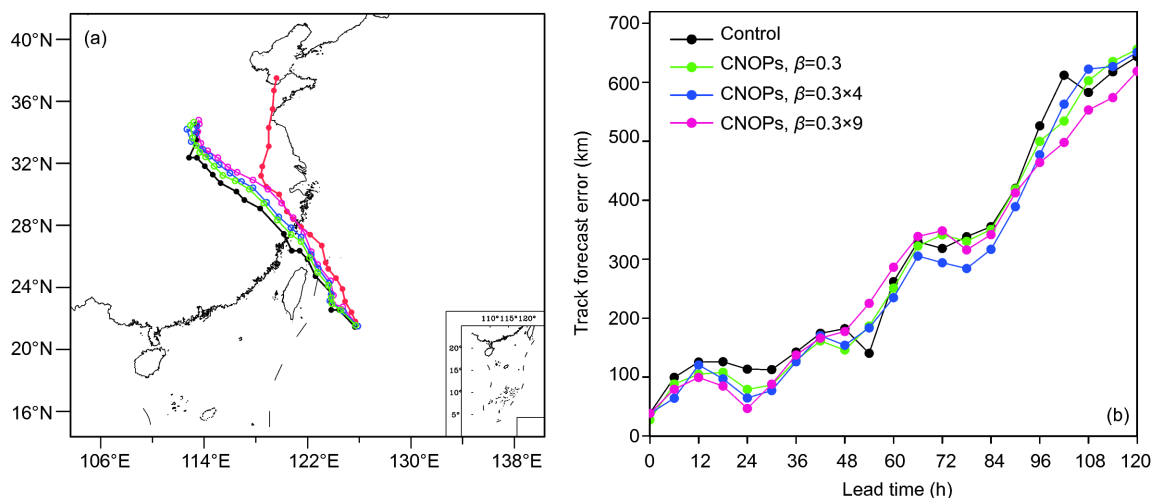


Figure 4 Ensemble forecast results for STY Matsa in 2005 with the orthogonal CNOPs method for different initial perturbation amplitudes. (a) Control forecast (black line), observed (red line), and CNOPs-ensemble mean ($\beta=0.3$, green line; $\beta=0.3 \times 4$, blue line; and $\beta=0.3 \times 9$, carmine line) tracks. (b) Evolution of the track forecast errors for the control forecast (black line) and CNOPs-ensemble mean ($\beta=0.3$, green line; $\beta=0.3 \times 4$, blue line; and $\beta=0.3 \times 9$, carmine line). Here, the CNOPs-ensemble mean is the equally weighted ensemble mean by averaging all of the ensemble members.

Table 1 The average track forecast error (km) over a 5-day forecast period for the control forecast and the equally weighted CNOPs-ensemble mean by averaging all of the ensemble members for different amplitudes of β

Method	β		
	0.3	0.3×4	0.3×9
Control	311.04	311.04	311.04
CNOPs	301.55	291.01	292.71

forecast errors of the CNOPs-ensemble means for $\beta=0.3 \times 4$ and $\beta=0.3 \times 9$ are much smaller than the track forecast errors for the control forecast. Table 2 gives the average track forecast errors over 5 days for the control forecast and the CNOPs-ensemble means for different amplitudes of β . The results show that the average track forecast error of the CNOPs-ensemble mean decreases with β , and the CNOPs-ensemble mean has the smallest average track forecast error when $\beta=0.3 \times 9$.

Effectively obtaining more reasonable and accurate analyses with the information provided by the ensemble members is very important for the success of the ensemble forecasts. If the treatment is not suitable, we may obtain an ensemble mean forecast that performs worse than a single and deterministic forecast. Zhang and Krishnamurti (1997) showed that the selected ensemble mean could further improve the equally weighted ensemble mean. Hence, when obtaining an ensemble mean, we should selectively obtain the ensemble mean based on the performances of different forecast members rather than using the simply equally weighted ensemble mean to avoid the negative impacts of anomalously perturbed forecasts with small probabilities on the skill of the whole forecast. Of course, information for anomalously perturbed forecasts requires the attention of

forecasters because anomalously perturbed forecasts result in extreme probabilities. If we can set different weights for different forecast members according to the performance of each forecast member and obtain the unequally weighted ensemble mean, we may further improve the forecast skills of the ensemble mean. This is the topic of our future research. In this paper, we concentrate on whether the orthogonal CNOPs method could effectively improve typhoon control forecasts.

4.3 Comparison of ensemble forecast skills with different methods

In section 4.2, we analyzed the influence of varying amplitudes of orthogonal CNOPs on the ensemble forecast skill. The results showed that the orthogonal CNOPs method could improve the control forecast. However, does it perform better than the orthogonal SVs method and CNOP+SVs method? To answer this question, we adopt the orthogonal SVs method and CNOP+SVs method here to conduct ensemble forecast experiments and compare the ensemble forecast skills of different methods. According to section 4.2, we know that the orthogonal CNOPs method obtains the highest ensemble forecast skill when $\beta=0.3 \times 9$. Hence, we first set $\beta=0.3 \times 9$ to produce 5 initial perturbations for each method and then compare the forecast skills of the ensemble mean tracks for different methods. For simplicity, we call the ensemble means obtained by the orthogonal CNOPs method, orthogonal SVs method and CNOP+SVs method the CNOPs-ensemble mean, SVs-ensemble mean and CNOP+SVs-ensemble mean, respectively. It is noted that there is not an anomalously perturbed forecast member for the orthogonal SVs method. Therefore, the SVs-ensemble mean is

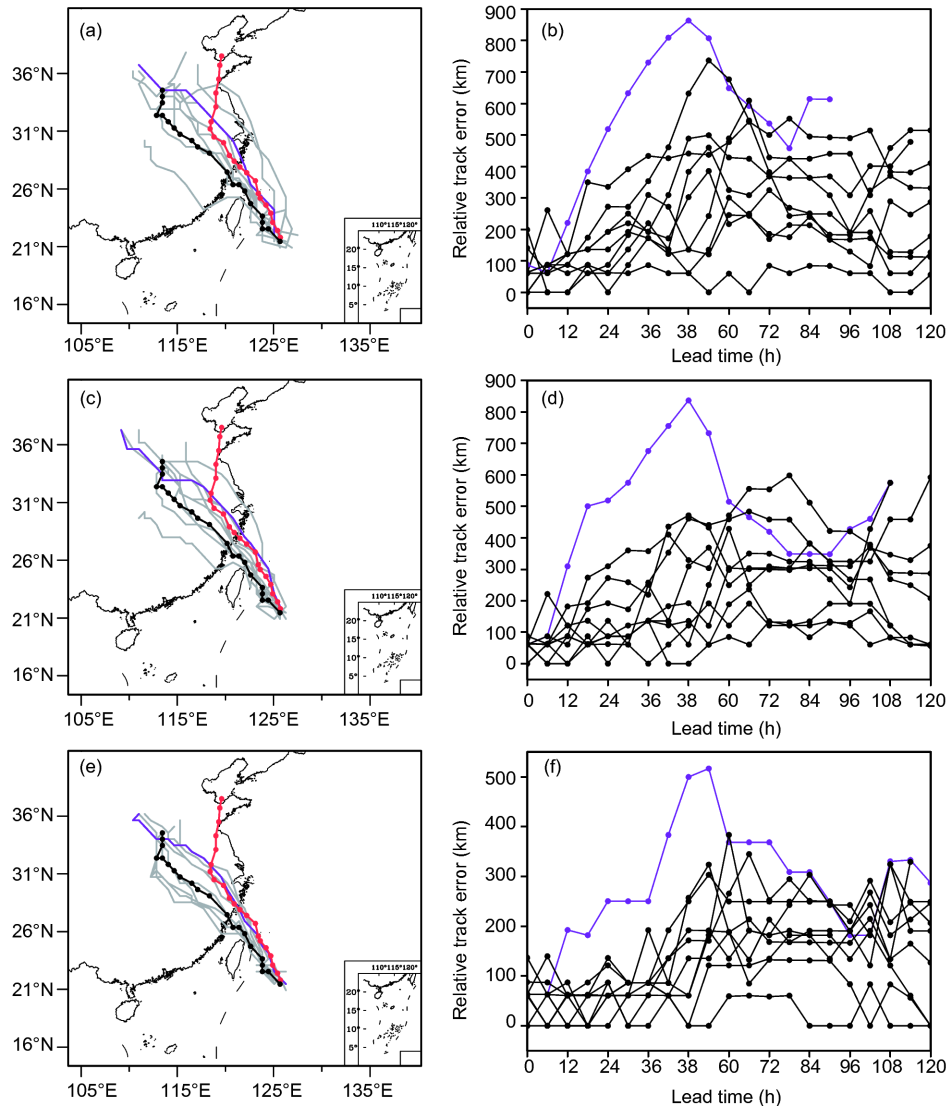


Figure 5 The ensemble forecast results with the orthogonal CNOPs method for STY Matsa in 2005 when $\beta=0.3 \times 9$ ((a) and (b)), $\beta=0.3 \times 4$ ((c) and (d)), and $\beta=0.3$ ((e) and (f)). (a), (c), and (e) Perturbed forecast (gray lines), observed (red line), control forecast (black line), and anomalously perturbed forecast (purple line) typhoon tracks. (b), (d), (f) Evolution of track errors (km) for the perturbed forecasts compared to the control forecast (black lines), where the purple line corresponds to the anomalously perturbed forecast.

Table 2 The average track forecast errors (km) over 5 days for the control forecast and the ensemble means for different methods and amplitudes of β

Method	β		
	0.3	0.3×4	0.3×9
Control	311.04	311.04	311.04
CNOPs	294.63	269.45	269.03
SVs	308.43	302.24	295.48
CNOP+SVs	313.43	324.76	291.18

the equally weighted mean obtained by averaging all of the ensemble members. However, for the CNOP+SVs method, because the first CNOP considers the influence of non-linearity, the corresponding perturbed forecast is always the abnormal forecast relative to the ensemble members gener-

ated by the orthogonal SVs method. If we abandon the anomalously perturbed forecast and obtain the ensemble mean with the other ensemble members, the significance of the CNOP+SVs method is lost. Therefore, the CNOP+SVs-ensemble mean is the equally weighted mean obtained by averaging over all ensemble members.

Figure 7a gives the forecast tracks of the ensemble means for different methods when $\beta=0.3 \times 9$. The results show that the CNOPs-ensemble mean successfully forecasts the land-fall location of STY Matsa, while the landfall locations for the ensemble means using the other methods are very close to the landfall location of the control forecast and far away from the observed landfall location. Regarding the whole forecast track, the CNOPs-ensemble mean is the closest to the observed track. **Figure 7b** gives the evolution of the track

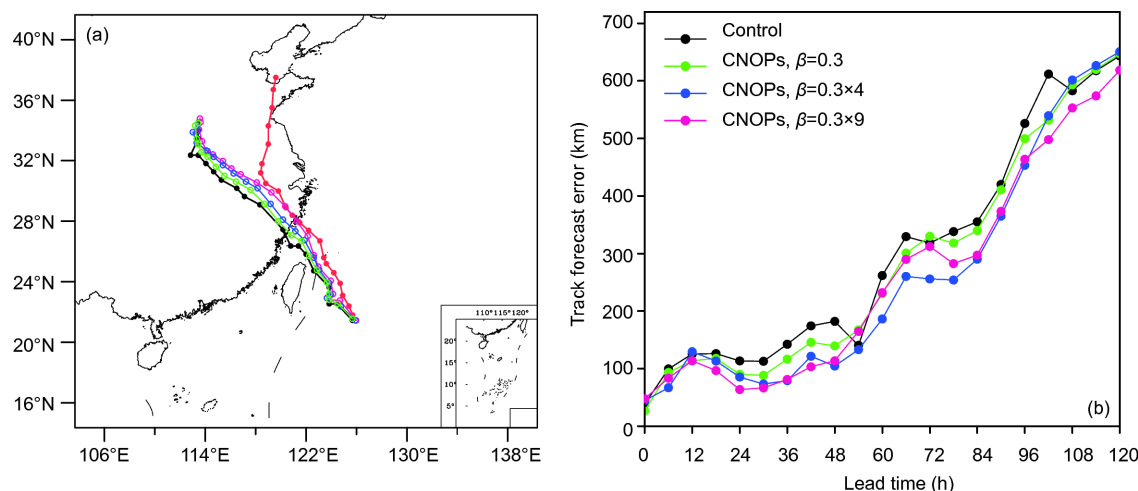


Figure 6 Ensemble forecast results for STY Matsa in 2005 with the orthogonal CNOPs method for different initial perturbation amplitudes. (a) Control forecast (black line), observed (red line), and CNOPs-ensemble mean ($\beta=0.3$, green line; $\beta=0.3\times 4$, blue line; and $\beta=0.3\times 9$, carmine line) tracks. (b) Evolution of track forecast errors for the control forecast (black line) and CNOPs-ensemble mean ($\beta=0.3$, green line; $\beta=0.3\times 4$, blue line; and $\beta=0.3\times 9$, carmine line). Here, the CNOPs-ensemble mean is the equally weighted ensemble mean by averaging all of the ensemble members that are not abnormal.

forecast errors for the ensemble means using different methods when $\beta=0.3\times 9$. The results show that the orthogonal CNOPs method improves the forecast of the typhoon track at its largest extent throughout the whole forecast period. Specifically, the 5-day average track forecast error for the CNOPs-ensemble mean (see Table 2) is 42.01 km smaller than that of the control forecast, with an improvement of 13.51%.

Since the amplitudes of the initial perturbations can affect the ensemble forecast skills, we compared the ensemble forecast skills of the different methods for different values of β . The results are shown in Figure 7 and Table 2. All of the results show that when $\beta=0.3$ and $\beta=0.3\times 4$, the CNOP+SVs method not only has a lower forecast skill than the orthogonal SVs method but also performs worse than the control forecast; when $\beta=0.3\times 9$, the CNOP+SVs method has a higher forecast skill than the orthogonal SVs method, but the improvement is small. Hence, only replacing the LSV with the first CNOP while maintaining the other SVs may not contribute to a higher forecast skill compared to the orthogonal SVs method. Furthermore, when compared with the SVs-ensemble mean and CNOP+SVs-ensemble mean, the CNOPs-ensemble mean has the highest forecast skill for a given value of β which, in some ways, validates the superiority of the orthogonal CNOPs method. It is noted that STY Matsa in 2005 is a strong event. These results may validate the deduction that Duan and Huo (2016) made (i.e., the orthogonal CNOPs method has a higher forecast skill when forecasting strong events).

5. Summary and discussion

In this study, we first compare the ensemble forecast skills of

the orthogonal CNOPs method, orthogonal SVs method and CNOP+SVs method with the Lorenz-96 model. The results show that the CNOP+SVs method has a higher forecast skill than the orthogonal SVs method, but the orthogonal CNOPs method has the highest ensemble forecast skill. This means that only replacing the LSV with the first CNOP (but maintaining the other SVs) still has linear limitations; fully considering the influence of nonlinearity is important when generating fast-growing ensemble initial perturbations.

To test the usefulness of orthogonal CNOPs in typhoon ensemble forecasts, we applied the orthogonal CNOPs method in the typhoon ensemble forecasts with the MM5. Specifically, we conducted typhoon track ensemble forecast experiments for STY Matsa in 2005 and compared the ensemble forecast skills of the orthogonal CNOPs method, orthogonal SVs method and CNOP+SVs method.

The spatial structures of orthogonal CNOPs and orthogonal SVs show that there are large differences between orthogonal CNOPs and SVs, which reflects the influence of nonlinear physical processes on the forecast results. Orthogonal CNOPs have a wider spatial distribution and better describe uncertainties in the initial analysis field, which may help to achieve higher ensemble forecast skills. In addition to the first CNOP, other CNOPs also have large differences compared with SVs. This suggests that fully considering the influence of nonlinearity may be necessary when generating fast-growing ensemble initial perturbations.

We studied the influence of the amplitudes of initial perturbations on the ensemble forecast skill of the orthogonal CNOPs method. The results show that when the ensemble mean is defined as an equally weighted mean obtained by averaging all ensemble members, the orthogonal CNOPs method has a higher forecast skill than the control forecast

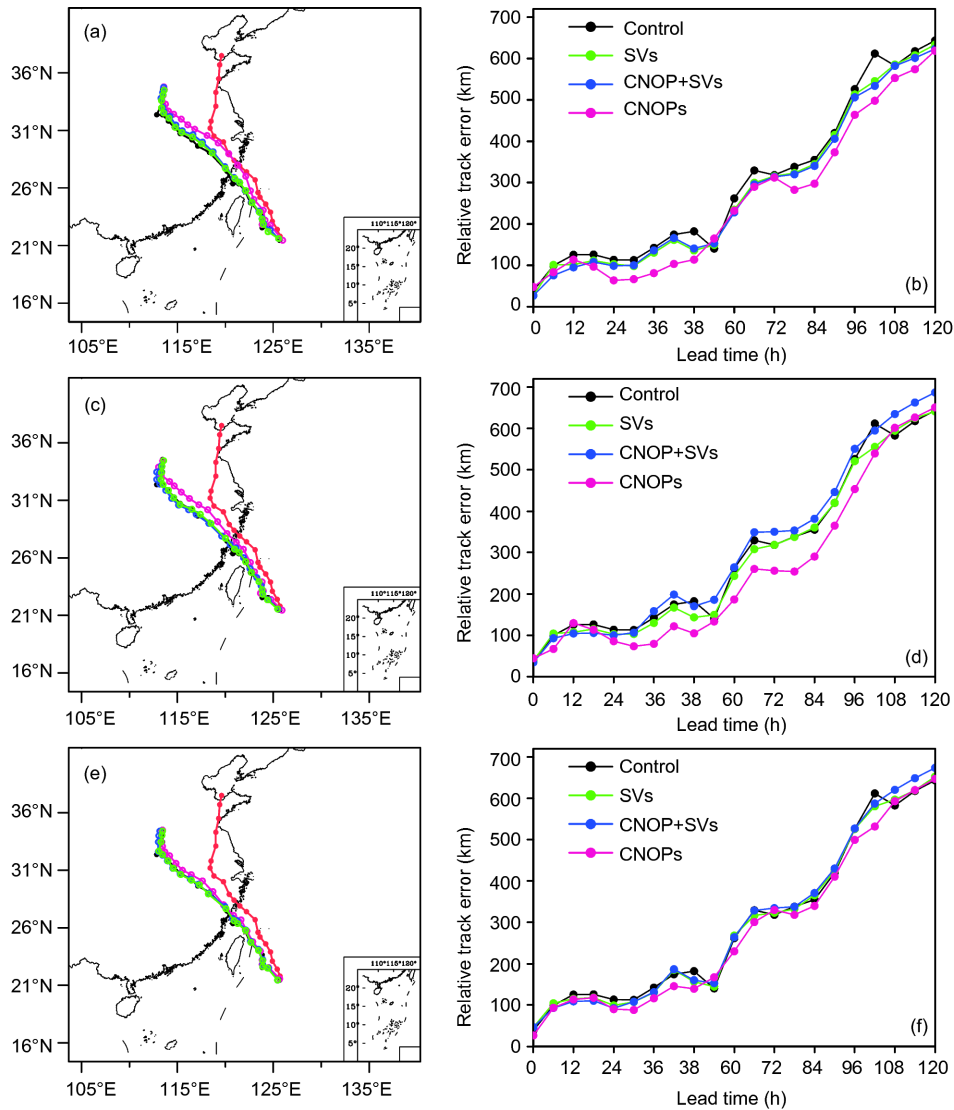


Figure 7 Ensemble forecast results with different methods for STY Matsa in 2005 when $\beta=0.3\times 9$ ((a) and (b)), $\beta=0.3\times 4$ ((c) and (d)) and $\beta=0.3$ ((e) and (f)). (a), (c), and (e) Tracks from the observations (red line), control forecast (black line), CNOPs-ensemble mean (carmine line), SVs-ensemble mean (green line) and CNOP+SVs-ensemble mean (blue line). (b), (d), and (f) Evolution of the track forecast errors for the control forecast (black line), CNOPs-ensemble mean (carmine line), SVs-ensemble mean (green line), and CNOP+SVs-ensemble mean (blue line).

for given amplitudes of the initial perturbations. However, the CNOPs-ensemble mean has a comparable track forecast error when $\beta=0.3\times 4$ and $\beta=0.3\times 9$, which does not coincide with the CNOPs-ensemble mean tracks. Further investigation shows that there is an anomalously perturbed forecast member among the ensemble members, which is obtained by the orthogonal CNOPs method. To avoid the negative impact of the anomalously perturbed forecast member on the ensemble mean, we compute the ensemble mean as an equally weighted mean by averaging ensemble members that are not abnormal and compare the forecast skills of the new ensemble mean and the equally weighted ensemble mean by averaging over all ensemble members. The results show that the new ensemble mean has a higher forecast skill. Of course, information regarding the anomalously perturbed

forecasts requires the attention of forecasters because anomalously perturbed forecasts result in extreme probabilities. If we can set different weights for different forecast members according to the performance of each forecast member and obtain the unequally weighted ensemble mean, then we may further improve the forecast skill of the ensemble mean. This is an area we need to pay more attention to in future research.

Finally, we compare the ensemble forecast skills of the orthogonal CNOPs method, orthogonal SVs method and CNOP+SVs method. The results show that only replacing the LSV with the first CNOP (but maintaining the other SVs) may not contribute to a higher forecast skill compared with the orthogonal SVs method. However, if we adopt the orthogonal CNOPs method for ensemble forecasts, we may

obtain the highest ensemble forecast skill. This indicates that fully considering the influence of nonlinear physical processes on forecast results may be very important when generating fast-growing ensemble initial perturbations. All of the results consistently verify that the orthogonal CNOPs method may be a potentially new ensemble forecasting method.

Jiang et al. (2009) showed that ensemble forecast skills of different initial ensemble perturbations (e.g., SVs and CNOP+SVs) depend on the type of analysis error. They pointed out that understanding the information for the analysis error and adopting a suitable initial ensemble perturbation method are very important. Their results show that the determination of the constraint value β needs further discussion. Generally, β should be an estimation of the analysis-error variance. However, practical implementation indicates that the determination of β should be based on experience through a large number of numerical experiments. This issue needs our attention in the future.

The norm used at the initial time and final time to compute the SVs and CNOPs is the dry total energy norm in this paper. However, the correct choice is indeed the analysis-error covariance norm (Ehrendorfer and Tribbia, 1997). Gelaro et al. (2002) found that the distribution of leading analysis-error covariance SVs is consistent with the expected distribution of analysis errors. Hamill et al. (2003) further showed that operational ensemble forecasts based on total energy SVs could be improved by changing total energy SVs to be flow-dependent analysis-error covariance SVs. When focusing on tropical cyclone track prediction, the norms that are more directly related to typhoon movement are of interest. Due to the fact that SVs and CNOPs are sensitive to the chosen norm, it is also of interest to compare the difference in orthogonal CNOPs and SVs using different norms at the initial time and examine how sensitive the performances of the orthogonal CNOPs method, SVs method and CNOP+SVs method in typhoon ensemble forecasts are to the norms. Mu et al. (2009) showed that the structures of CNOPs differed much from those for first singular vectors (FSVs) depending on the constraint, metric and basic state. Wang et al. (2011) indicated that the background-error covariance metric at the initial time and the total energy norm at the final time are reasonable choices for the computation of CNOPs. They pointed out that the benefit of employing a background-error covariance is that statistical uncertainties in the background field are also considered, which helps the CNOP represent the structure that is likely to occur statistically and initial perturbation that has the largest nonlinear evolution. The information of analysis or background errors is important for ensemble forecasts. The impact of taking the analysis-error or background-error covariance metric as the initial norm to compute SVs and CNOPs and using the information of analysis or background errors to constraint the

SVs and CNOPs will be studied in our future work.

Here, one case (i.e., STY Matsa) is chosen to preliminarily study the effectiveness of the orthogonal CNOPs method in typhoon ensemble forecasts. However, since analysis error information, SVs and CNOPs are flow-dependent, a few cases are required to draw a solid conclusion. Moreover, the forecast of typhoon intensity has important significance for early warnings and decisions on typhoons. Therefore, for more typhoons and the forecasting of typhoon intensity, how will the orthogonal CNOPs method perform? This issue will be considered in our future work. Two other issues in this paper are the coarse resolution and how the results shown are affected by resolution, which will also be studied in our future work. Furthermore, there are many other ensemble forecasting methods that generate initial perturbations, such as the bred vector method (Toth and Kalnay, 1993) and the ensemble Kalman filter method (EnKF; Evensen, 1994). In our future work, we will further compare the ensemble forecast skills of the orthogonal CNOPs method and other methods to verify the superiority of the orthogonal CNOPs method and improve the details of the orthogonal CNOPs method. In addition, to advance the success of operational ensemble forecasts of tropical cyclones, it is crucial to investigate the efficiency of optimization algorithms when solving CNOPs since we are required to obtain the initial perturbations as soon as possible for operational ensemble forecasts.

Acknowledgements The FNL data used in this study can be obtained from <http://rda.ucar.edu/datasets/ds083.2/>. The historical tropical cyclone data are available at http://tcdata.typhoon.org.cn/en/zjljsjj_zlhq.html. This work was sponsored by the National Natural Science Foundation of China (Grant Nos. 41525017 & 41475100), the National Programme on Global Change and Air-Sea Interaction (Grant No. GASL-IPOVAI-06), and the GRAPES Development Program of China Meteorological Administration (Grant No. GRAPES-FZZX-2018).

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(Responsible editor: Zhiyong MENG)